GENERALIZATION OF A THEOREM OF JENSEN CONCERNING CONVOLUTIONS

By H. W. Gould

1. **Introduction.** In two previous papers [1], [2] we have discussed certain convolution identities involving the coefficients of a class of power series. The general setting for our considerations is as follows.

Let it be supposed that

$$x^\alpha = \sum_{k=0}^{\infty} C_k(\alpha, \beta) z^k, \tag{1.1}$$

and

$$x^\alpha \cdot g(x, \beta) = \sum_{k=0}^{\infty} G_k(\alpha, \beta) z^k, \tag{1.2}$$

are convergent power series where

$$G_k(\alpha, \beta) = \frac{\alpha + \beta k}{\alpha} C_k(\alpha, \beta), \tag{1.3}$$

and

$$z = \frac{f(x)}{x^\beta}. \tag{1.4}$$

Then it is a routine calculation to use the relations

$$x^\alpha \cdot x^\gamma = x^{\alpha+\gamma},$$

$$x^\alpha \cdot x^\gamma g(x, \beta) = x^{\alpha+\gamma} g(x, \beta),$$

$$x^\alpha g(x, \beta) \cdot x^\gamma g(x, \beta) = x^{\alpha+\delta} g(x, \beta) \cdot x^{\alpha-\delta} g(x, \beta),$$

and term-by-term differentiation (in the case of (1.8)) to verify the following identities:

$$\sum_{k=0}^{n} C_k(\alpha, \beta) C_{n-k}(\gamma, \beta) = C_n(\alpha + \gamma, \beta), \tag{1.5}$$

$$\sum_{k=0}^{n} C_k(\alpha, \beta) G_{n-k}(\gamma, \beta) = G_n(\alpha + \gamma, \beta), \tag{1.6}$$

$$\sum_{k=0}^{n} G_k(\alpha, \beta) G_{n-k}(\gamma, \beta) = \sum_{k=0}^{n} G_k(\alpha + \delta, \beta) G_{n-k}(\alpha - \delta, \beta), \tag{1.7}$$