TREE-LIKE CONTINUA AND QUASI-COMPLEXES

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1. **Introduction.** In [3], Dyer showed that every snake-line continuum is a quasi-complex (see [1] or [5] for the definition of a quasi-complex). In this paper we show that Dyer's result does not extend to tree-like continua. After two preliminary theorems we prove a lemma which is crucial in showing certain tree-like continua are not quasi-complexes (see Lemma 1 below). As a final result we show that any tree-like continuum is the intersection of a countable number of tree-like continua that are quasi-complexes.

The present paper is an exposition of some of the results announced in [2]. It appears there is overlapping with some results announced by Keisler in [4].

2. **Some definitions.** By a tree we mean a finite acyclic 1-dimensional complex. If X is a compact metric space, by a cover of X we mean a covering of X by a finite number of open sets. The nerve $N(\{U_i\})$ (abbreviated $N(U)$) of a cover $\{U_i\}$ of X is the complex determined in the usual manner from the intersection pattern of the $\{U_i\}$.

A tree-like continuum is a compact metric space with a cofinal sequence of covers whose nerves are trees. Since tree-like continua are the only spaces discussed in this paper and since we are only interested in the covers whose nerves are trees, the terms 'space' and 'cover' will be used in this restricted sense.

Suppose X has a metric ρ. Then given a cover $\{U_i\}$ there is a positive number (which we will denote by $L(\{U_i\})$) such that for any cover $\{V_i\}$ with mesh $\{V_i\} < L(\{U_i\})$ each $V_i \in \{V_i\}$ is contained in some $U_j \in \{U_i\}$. If also for some U_j with $V_i \subset U_j$ we have $\bar{V}_i \subset U_j$($\bar{V}_i$ is the closure of V_i), we say that $\{V_i\}$ is a refinement of $\{U_i\}$.

If p and q are vertices of a tree K, $[p, q]$ will denote the arc in K connecting p with q. $[p, q]$ will also be used to denote the integral 1-chain which is 1 on the simplexes in the arc $[p, q]$ oriented from p to q and 0 otherwise. Where necessary, the modifiers 'arc' and '1-chain' will be used to distinguish the two notions.

The integral chain group of K will be denoted by $C(K)$. $C^{(0)}(K)$ and $C^{(1)}(K)$ will be the subgroups of the 0-chains and 1-chains respectively. If K is the nerve of a cover of X and C is a chain of $C(K)$, by $|\ C\ |$ we mean the carrier of C (see Brahana [1; 954] under (c); notice our notation is different from that in [1]). If σ is an oriented simplex of K, by a simplex chain σ we mean the chain which is 1 on σ, -1 on $-\sigma$ and 0 otherwise. Where confusion is unlikely to arise, the expression 'simplex chain' will be dropped.

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