POLYNOMIAL 3-COCYCLES OVER FIELDS OF CHARACTERISTIC p

By Robert Heaton

1. **Introduction.** A polynomial $f(x, y, z)$ over a field K is a 3-cocycle in the sense of [5] if we have the identity

$$f(y, z, w) - f(x + y, z, w) + f(x, y + z, w) - f(x, y, z + w) + f(x, y, z) = 0. \tag{1}$$

Further $f(x, y, z)$ is the coboundary $\delta g(x, y, z)$ of a polynomial $g(x, y) = \sum_{\mu,\nu} a_{\mu\nu} x^\mu y^\nu$ over K when

$$f(x, y, z) = \delta g(x, y, z) = g(y, z) - g(x + y, z) + g(x, y + z) - g(x, y) = \sum_{\mu,\nu} a_{\mu\nu}(\Delta_\mu(x, y)z^\nu - \Delta_\nu(y, z)x^\mu) \tag{2}$$

where

$$\Delta_\nu(x, y) = x^\nu + y^\nu - (x + y)^\nu. \tag{3}$$

It was shown in [5] that every polynomial n-cocycle over a field of characteristic 0 is the coboundary of a polynomial, and results were also obtained for $n = 1, 2$ in the case of characteristic p. One finds that the polynomial 1-cocycles are just additive polynomials about which there is an extensive theory, see [1;7], and the 2-cocycles are cohomologous to a non-symmetric biadditive polynomial and a linear combination of polynomials $\varphi_p(x, y)^{p^\nu}$ where as in [5] $\varphi_p(x, y)$ is the image of $p^{-1}\Delta_p(x, y)$ under the natural homomorphism of $Z[x, y]$ onto $Z/(p)[x, y]$, Z being the ring of integers. This paper is intended to extend the results to 3-cocycles. Besides their intrinsic interest, the polynomial 3-cocycles could be used to give a description of the similarity classes of Q-kernels with center G (see [3]) when Q and G are additive groups of Galois fields, one being an algebraic extension of the other.

2. **Preliminaries.** By a shuffle of x through x, y, z, w, we shall mean the resulting permutations x, y, z, w; y, x, z, w; y, z, x, w; and y, z, w, x. If $f(x, y, z)$ is a 3-cocycle, a shuffle of x applied to (1) yields the four identities $\delta f(x, y, z, w) = 0$, $-\delta f(y, x, z, w) = 0$, $\delta f(y, z, x, w) = 0$, and $-\delta f(y, z, w, x) = 0$. Adding these identities, grouping, and letting

$$F(x, y, z) = f(x, y, z) - f(y, x, z) + f(y, z, x), \tag{4}$$

gives the identity

$$F(x, z, w) - F(x, y + z, w) + F(x, y, z + w) - F(x, y, z) = 0. \tag{5}$$

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