CONCERNING A CERTAIN COLLECTION OF SPIRALS IN THE PLANE

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Let Σ be a family such that G belongs to Σ if and only if G is a collection of mutually exclusive arcs in the plane such that there exist a straight line L and a side D of L such that (1) each arc of G has one endpoint on L and lies, except for this point, in D, (2) G^* is bounded, and (3) if g is an arc in G and A is the endpoint of g not on L, then g spirals down on A. The result of this paper is the following: *There exists a collection* Γ *in* Σ *such that the point set* M, *to which* P *belongs if and only if some arc of* Γ *spirals down on* P, *is an arc.* (For a definition of a spiral, and of spiralling down, see [1].)

1. Notation and terminology. If G is a collection of sets, G^* denotes the union of the sets of G. If M is a point set, $\bar{M}$ denotes the closure of M. If x is a point or a set, $\{x\}$ denotes the set whose only element is x. "Interval", used without further qualification, means "straight-line interval"; similarly for "segment". The straight-line interval AB is often denoted by AB.

The statement that the arc M is an *A-arc* means that there exist two points P and Q and a sequence of points $P_0, P_1, P_2, \cdots$ such that (1) P is P_0, P_1 is vertically below P_0, and P_2 is on the horizontal line through P_1 and to the left of P_1, (2) if n is a positive integer, then the angle $P_{n-1}P_nP_{n+1}$ is a right angle and has P_{n+2} in its interior, (3) there is a horizontal line h below P_0 such that for each positive integer n, P_n is below h, (4) the sequence $P_0, P_1, P_2, \cdots$ converges to Q, and (5) M is $\bigcup_{n=0}^{\infty} P_nP_{n+1} \cup \{Q\}$. The points P and Q are called the upper and lower endpoints, respectively, of M. Note that M spirals down on Q.

The statement that the domain D is an *A-domain* means that there exist three points P, Q, and R, such that PQ is horizontal, and two A-arcs, PR with upper endpoint P and QR with upper endpoint Q, having only R in common, such that D is the interior of the simple closed curve $PR \cup QR \cup PQ$. Let $C(D)$ be a set such that x belongs to $C(D)$ if and only if for some segment t, either vertical or horizontal, with one endpoint on the arc PR, the other on the arc QR, and lying wholly in D, x is the length of t. Let $W(D)$ denote the least upper bound of the number set $C(D)$. The straight-line interval PQ and $\{R\}$ are called the upper and lower ends, respectively, of D.

The statement that the arc K is a *B-arc* means that there exist an A-arc PQ whose upper endpoint is P and a point X of the segment PQ of the arc PQ such

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