ON TREE-LIKE CONTINUA AND IRREDUCIBILITY

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1. **Introduction.** A *chain* is a collection of open sets $(d_1, \cdots, d_k)$ such that $d_i \cdot d_j \neq 0$ if and only if $| i - j | \leq 1$. A collection of sets G is said to be *coherent* if for every proper subcollection G' contained in G, there is an element of G' which intersects an element of $G - G'$. A *circular chain* will be defined to be a collection of open sets which is the union of two chains $(d_1, \cdots, d_k)$ and (e) so that $d_i \cdot e \neq 0$ if and only if $i = 1$ or $i = k$ and $k \geq 2$. By this definition, for the sake of brevity, a collection consisting of three open sets whose common intersection is non-empty is a circular chain, although it is usually specified that a circular chain is of order two. A *tree-chain* is a finite coherent collection of open sets which contains no circular chains. In a metric space if each element of a tree-chain has diameter less than ϵ, then the collection will be called an ϵ-tree-chain, or we shall say that the *mesh* of the collection is less than ϵ. A *tree-like continuum* (*snake-like continuum*) is a compact metric continuum which, for each positive number ϵ, can be covered by an ϵ-tree-chain (ϵ-chain). This terminology was introduced by R. H. Bing. Theorems 1 and 3 are generalizations for tree-like continua of theorems proved by R. H. Bing for snake-like continua.

In discussing tree-chains it is convenient to use the following terminology. If we think of a coherent collection of sets as being connected by the property of set intersection, then such a collection may be called *star-connected* since for any two elements of the collection, s and t, there is a chain $(s, \cdots, t)$ contained in the collection; if the chain has k links, t is in the k-th star of s with respect to the collection. In the same vein, if G is a collection of sets and g is in G, the *star-component* of g with respect to G will be the maximal coherent subcollection in G containing g. If d is a link of the tree-chain T and d intersects at most one other element of T, then d will be called an *end link* of T. Furthermore, a tree-chain will be called *k-branched* or said to have *k branches* provided that it has exactly k end links.

The following elementary properties of tree-chains will be used throughout.

1) If d and d' are links in a tree-chain T, then there is a unique chain in T whose end links consist of d and d'.

2) Let T be a k-branched tree-chain and $(d_1, \cdots, d_k)$ its set of end links. If D_{1j} is the chain in T linking d_1 to d_j, $j = 2, \cdots, k$, then $T = D_{12} + \cdots + D_{1k}$.

3) If T is a tree-chain and $C = (c_1, \cdots, c_k)$ is a collection of links of T, $k \geq 3$, such that no three distinct links from C lie in a chain in T, then T has n end links, $n \geq k$.

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