BERNOULLI AND EULER NUMBERS AND ORTHOGONAL POLYNOMIALS

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1. **Introduction.** Touchard [7] has constructed a set of polynomials $\Omega_n(z)$ such that

$$B^r\Omega_n(B) = K_n\delta_{rn} \qquad (0 \le r \le n), \tag{1.1}$$

where after expansion of the left member B^n is replaced by B_n,

$$e^{Bx} = \sum_{n=0}^{\infty} B_n \frac{x^n}{n!} = \frac{x}{e^x - 1}, \tag{1.2}$$

and

$$K_n = \frac{(-1)^n}{2n+1}\frac{1}{2^n}\frac{(n!)^4}{\{1.3.5 \cdots (2n-1)\}^2}. \tag{1.3}$$

Also $\Omega_n(z)$ satisfies

$$\Omega_{n+1}(z) = (2z+1)\Omega_n(z) + \frac{n^4}{4n^2-1}\Omega_{n-1}(z). \tag{1.4}$$

Using (1.4) Wyman and Moser [10] showed that

$$\Omega_n(z) = 2^n \cdot n!\binom{2n}{n}^{-1} \sum_{2r \le n} \binom{2z+n-2r}{n-2r}\binom{z}{r}^2, \tag{1.5}$$

and by means of (1.5) the writer [3] showed that

$$\Omega_n(z) = (-2)^n n!\binom{2n}{n}^{-1} F_n(2z+1), \tag{1.6}$$

where $F_n(z)$ is Bateman's polynomial [1]

$$F_n(z) = {}_3F_2\left[\begin{matrix} -n, n+1, \frac{1}{2}(1+z) \\ 1, 1 \end{matrix}\right]. \tag{1.7}$$

In the present paper we show first that Touchard's result (1.1) can be extended to the numbers

$$\beta_n = \beta_n(\lambda) = \frac{B_{n+1}(\lambda) - B_{n+1}}{(n+1)\lambda}, \tag{1.8}$$

where $B_n(\lambda)$ is the Bernoulli polynomial defined by

$$\frac{ze^z}{e^z - 1} = \sum_{n=0}^{\infty} B_n(\lambda)\frac{z^n}{n!}.$$

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