# REGULAR MAPPINGS AND THE SPACE OF HOMEOMORPHISMS ON A 2-MANIFOLD

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1. **Introduction**. That the space, SO(3) of rotations of the 2-sphere $S^2$ is a deformation retract of the space $H(S^2)$ of orientation preserving homeomorphisms of $S^2$ onto itself was proved by H. Kneser in 1926 [5]. This contains one of the results announced by the authors when presenting the present paper in abstract form. That Kneser's paper contains this theorem does not seem to be well known, and we are indebted to Professor H. Samelson for pointing this out to us.

It is not difficult to see that $SO(3)$ is homeomorphic to real projective 3-space, $P^3$. It then follows directly from Kneser's result that $\pi_n(P^3) = \pi_n(H(S^2))$ for each $n$. Therefore $\pi_1(H(S^2)) = Z_2$ and $\pi_n(H(S^2)) = \pi_n(S^3)$, $n > 1$, since $S^3$ is a covering space of $P^3$, and the Hopf fibring of $S^3$ gives $\pi_n(H(S^2)) = \pi_n(S^2)$ for $n > 2$.

It can be proved directly and easily from the facts that $H(S^2)$ is homogeneous and $SO(3)$ is a deformation retract of $H(S^2)$ that $H(S^2)$ is locally contractible. This generalizes results in [2] and [7]. In §3 of this paper we shall give a different proof of this fact, using some of the techniques described by Kneser. The merit in this proof lies in the fact that it holds for compact 2-manifolds in general. We shall then be able to prove, under certain restrictive conditions on $Y$, that if $f$ is a 0-regular mapping of a metric space $X$ onto a space $Y$ such that each inverse under $f$ is homeomorphic to a compact 2-manifold, $M$, then $X$ is a locally trivial fibre space, $f$ corresponding to the projection map. This extends results in [3].

2. **Some remarks on conformal mappings.** Kneser made use of some results from conformal mapping theory. These results follow without difficulty from standard theorems. (See, for instance, Chapters IV and V of *Complex Analysis* by L. V. Ahlfors.) In this section, the word "plane" will be synonymous with "complex number plane." The coordinates used will be polar coordinates.

Suppose that $D$ is a disc in the plane (a simple closed curve plus its interior), and $a$ is a point in the interior of $D$. By the Riemann mapping theorem, there is a homeomorphism of $D$ onto the circular disc $E$ described by $|z| \leq 1$ which carries $a$ onto the origin and is conformal in the interior of $D$. This mapping is uniquely determined up to a rotation—i.e., it is uniquely determined by the image of a point in the boundary of $D$. Also, if $t$ is a closed subset of Bdry $(E)$

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