ON DENDRITIC SETS

By L. E. Ward, Jr.

1. **Introduction.** In earlier papers [5], [6], [7] the author characterized various types of acyclic continua in terms of their inherent order theoretic properties, and the present work is concerned with the same problem for connected and acyclic spaces which are not necessarily compact. Such spaces, if they are locally connected, are termed *dendritic*. It is not at present known just how "nice" a space must be in order to admit a nontrivial continuous partial order. However, Wallace [3] has shown that an indecomposable continuum does not have this property. Except for special cases, the problem of order theoretic characterization remains quite unsolved for cyclic continua.

Using the work of Nachbin [1], [2], who first considered partially ordered spaces, we attack the problem of compactifying dendritic spaces so that the compactification is dendritic. It is shown that such compactifications exist provided the space is convex in a certain sense.

2. **Dendritic spaces.** A Hausdorff space X is said to be *dendritic* if and only if it is connected, locally connected, and has the property that each two points can be separated in X by the omission of some third point. A compact dendritic space is a tree in the sense of [5]. In this paragraph we characterize dendritic spaces by order theoretic means, and this, in turn, leads to topological characterizations.

Our terminology is taken from [1], [2] and [4] through [8] inclusive. In brief, a partial order $\leq$ is *order dense* if, whenever $x < y$, there exists z such that $x < z < y$. We write $L(x) = \{a: a \leq x\}$ and $M(x) = \{a: x \leq a\}$. A partial order on a space X is *semicontinuous* if $L(x)$ and $M(x)$ are closed sets for each $x \in X$. It is *continuous* if its graph is a closed subset of $X \times X$. If, for each x and y in X such that $x \nleq y$, there exists $f \in C(X)$ (= the set of continuous and order preserving functions on X to the closed unit interval) such that $f(x) = 1$ and $f(y) = 0$, then $\leq$ is *strongly continuous*. It is easy to see that strong continuity implies continuity and that continuity implies semicontinuity.

A subset F of a partially ordered set is *increasing* (*decreasing*) provided $M(x) \subset F(L(x) \subset F)$ for each $x \in F$. The space X is *normally ordered* (with respect to the partial order $\leq$) provided that whenever F_0 and F_1 are, respectively, decreasing and increasing disjoint closed sets, there exists $f \in C(X)$ such that $f \mid F_i \equiv i (i = 0, 1)$. Nachbin [1] showed that if X is compact Hausdorff and $\leq$ is continuous, then X is normally ordered. We say that X is *completely order regular* if, for each closed set F and element $x \in X - F$, there exist

Received October 28, 1957; in revised form, January 13, 1958. Presented to the American Mathematical Society November 30, 1957.