THE SQUARE ROOT SET OF A SIMPLE BASIC SET OF POLYNOMIALS

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1. **Introduction.** The subject of basic sets of polynomials was introduced by J. M. Whittaker. For such a set $\{p_n(z)\}$, any polynomial, and in particular the polynomial z^n, admits a unique finite representation of the form:

$$z^n = \sum_i h_{ni} p_i(z) = \sum_i h_{ni} \sum_j p_{ij} z^j, \tag{1.1}$$

where (h_{ij}) is the operator set and (p_{ij}) the coefficient set associated with the set of polynomials $\{p_n(z)\}$ throughout this paper. Whittaker proved that the matrix $P = (p_{ij})$ has a unique two-sided reciprocal which is the matrix of operators $H = (h_{ij})$.

2. Nassif and Makar investigated in [3] the effectiveness and the order of the square root set of a simple *monic* set. Besides, they put some restrictions on the coefficients of the simple *monic* set, other than p_{nn}, in order to get their results stated in [3, Theorems 1 and 2]. They used in their investigation some results due to Boas [1] and Newns [4].

Here we investigate the effectiveness and the mode of increase of the square root set of *any* simple basic set of polynomials. The method of treatment in this paper is altogether different from that followed by Nassif and Makar. Regarding the effectiveness and the order, it will be shown in this paper that a simple basic set of polynomials and its square root set behave identically. We prove, in this investigation, the following two theorems:

Theorem 1. *Let $\{p_n(z)\}$ be a simple basic set of polynomials effective on $|z| = R$ where $R \geq R_0$; then its square root set is also effective on $|z| = R$ where $R \geq R_0$.*

Theorem 2. *A simple basic set of polynomials and its square root set are of the same order.*

Theorems 1 and 2 obviously imply the results of Nassif and Makar [3] in this connection.

3. **Notation and previous results.** For general terminology used in this paper the reader is referred to [2] and [5]. We use for our investigation the simple basic set of polynomials $\{p_n(z)\}$ and its square root set $\{u_n(z)\}$. The sets of operators and coefficients associated with $\{u_n(z)\}$ are (k_{ij}) and (u_{ij}) respectively. Since $\{u_n(z)\}$ is the square root set of $\{p_n(z)\}$, then we can write:

$$\{p_n(z)\} = \{u_n(z)\}\{u_n(z)\}. \tag{3.1}$$

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