GALOIS COHOMOLOGY OF ADDITIVE POLYNOMIAL AND N-TH POWER MAPPINGS OF FIELDS

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Introduction. Results. Call a field k a *Kaplansky field* if it has prime characteristic and if for every non-zero additive polynomial $f(x)$ with coefficients in k, $f(k^+) = k^+$. This condition is the part of "Kaplansky's Hypothesis" (see [6] or [8; 220]) which applies to the residue class field. For additive polynomials (a.p.) see [9] and the references given there.

Theorem 1. *A field of prime characteristic p is a Kaplansky field if and only if it has no algebraic extension of degree divisible by p.*

Call a field a *Brauer field* if it has no inseparable extension and has for each n at most one algebraic extension of degree n in any algebraic closure. Such fields were first discussed in [3; 61.]

Theorem 2. *Let k be a Brauer field of prime characteristic p. If k has no algebraic extension of degree p then $f(k^+) = k^+$ for every* a.p. *$f(x)$ over k. If k does have an algebraic extension of degree p, then if $f(x)$ is any* a.p. *over k and $\mathfrak{a}$ is its kernel ($=$ set of zeros) in k, then*

$$k^+/f(k^+) \cong \mathfrak{a}. \tag{1}$$

The second part of Theorem 2 coincides with Theorem 8 of [9]. J. T. Tate found a much more elegant proof than mine by using Galois cohomology; since he says he doesn't intend to publish it himself, I give it here. My proof of Theorem 1 also uses Tate's method; and this same method gives a useful result (Theorem 3) when applied to the *multiplicative* group of a Brauer field under n-th power mappings: this also I had proved before in a more complicated way. (The original proof is given in the Ph.D. thesis of T. H. M. Crampton, Indiana University, 1955.) The parallelism between additive and multiplicative things is perhaps more interesting than Theorem 3; but it does make possible a proof of the tamely ramified case of the existence theorem for local class fields which is even more elementary than previous ones since it eliminates all haggling about roots of unity.

If k is a Brauer field, let

$$\mathfrak{D}(k) = \prod_q q^{d(q)} \tag{2}$$

be the Steinitz-number defining the degree of the algebraic closure of k over k. It is a formal product, taken over all primes q, with $d(q)$ either a non-negative integer, or the symbol ∞: k has a cyclic extension of degree n if and only if n

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