GENERALIZED CONJUGATE NETS IN PROJECTIVE n-SPACE

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1. **Introduction.** In ordinary Euclidean space the intersections of triple systems of orthogonal surfaces are lines of curvature on the surfaces [4]. At a point x_0 of space, let the intersecting orthogonal surfaces of a triple system be denoted by S^1, S^2, S^3 and let the curves of intersection be taken to be the parametric curves so that the intersections of S^γ are the u^α-, u^β-curves in which (α, β, γ) represents any permutation of the set of integers (1, 2, 3). Since the u^α-, u^β-curves are lines of curvature of S^γ, they form the conjugate net in which the developables of the congruence of tangents to the u^γ-curves at points of S^γ intersect the surface S^γ. Thus any triple system of orthogonal surfaces defines by their intersections a triple system of conjugate nets. The property of conjugacy is a projective one. Therefore, the necessary and sufficient conditions that a triple system of surfaces intersect in conjugate nets on each surface of the system are projective conditions. Moreover, the condition that the developables of a congruence intersect a surface in curves of a conjugate net is likewise a projective one. B. J. Hollingsworth has studied these conditions, obtained some results, and extended these results to systems of generalized conjugate nets in projective n-space [5]. The present paper presents a revision of some of Hollingsworth's n-dimensional results together with certain extensions thereof.

THEOREM 1. *Let an n-ply system of hypersurfaces be defined such that n of them, $V^1, V^2, \cdots, V^n$ pass through an ordinary point x_0 of projective space, V^β denoting a u^β = constant hypersurface.* (Hereafter a Greek index will assume the range $1, 2, \cdots, n$, and a Latin index the range, $0, 1, 2, \cdots, n$.) *Let N^β denote the parametric net of $u^1, u^2, \cdots, u^{\beta-1}, u^{\beta+1}, \cdots, u^n$-curves of V^β, which will be called the opposing net of the hypercongruence G_β of u^β-tangents at points of V^β. Each of the hypercongruences $G_1, G_2, \cdots, G_n$ is conjugate* (see Note 1) *to its opposing net, if and only if each of $n - 2$ of them is conjugate to its opposing net.*

The second theorem of the paper is a projective generalization and n-dimensional extension of the following classical result due to Darboux [4]: *If two surfaces W_1, W_2 are parallel* (i.e., *the congruence of lines L normal to one surface is normal to the other*) *and if the curves of W_1 which correspond to the developables of the congruence L form a conjugate net, then the corresponding net on W_2 is likewise conjugate.*

For the n-dimensional projective generalization the concept of parallelism is replaced by that of perspectivity defined as follows: Two transversal hypersurfaces V_0 and V_z of a hypercongruence G_n in n-dimensional projective space will be said to be in perspective with respect to a fixed hyperplane π if and only

Received March 13, 1956.