WEIGHTED PARTITIONS FOR GENERAL MATRICES OVER A FINITE FIELD

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1. **Introduction.** Let $GF(q)$, where $q = p^n$ and p is a rational prime, denote the Galois field of order p^n. For $\alpha \in GF(q)$ put

$$(1.1) \qquad e(\alpha) = e^{2\pi i t(\alpha)/p}, \qquad t(\alpha) = \alpha + \alpha^p + \cdots + \alpha^{p^{n-1}}.$$

If $A = (\alpha_{ij})$ is a square matrix having elements in $GF(q)$, put $\sigma(A) = \sum_i \alpha_{ii}$. In this paper we consider the sum

$$(1.2) \qquad S = S(B, W, R, A) = \sum_{UAV=B} e\{\sigma(UW + RV)\},$$

where B, W, R, A, U, V are matrices over $GF(q)$, A is non singular of order m, B is square of order t, U', W, R' and V are $m \times t$ matrices, and the sum is over all pairs U, V satisfying the equation $UAV = B$. For $W = O$, $R = 0$, the sum reduces to $N_t^t(A, B)$, the number of solutions U, V of $UAV = B$, which is given in [2; §5]. We show (Theorem 4) that S can be expressed in terms of generalized Kloosterman sums defined for square matrices over $GF(q)$. In §7, a number of properties of these Kloosterman sums are given.

The analogous sum for symmetric matrices has been considered previously [3], and the skew case will appear in a later paper. The symmetric case is a generalization of a paper on weighted quadratic partitions by L. Carlitz [1].

2. **Notation and preliminaries.** Let $q = p^n$. Numbers of $GF(q)$ will be denoted by lower case Greek letters α, β, $\cdots$. Matrices with elements in $GF(q)$ will be denoted by italic capitals A, B, $\cdots$. $A(m, t)$ will denote a matrix of m rows and t columns and $A(m, t; r)$ a matrix of the same size having rank r. A' will denote the transpose of A. If $A = (\alpha_{ij})$ is square, then $\sigma(A) = \sum_i \alpha_{ii}$ is the *trace* of A. If $A = (\alpha_{ij})$ and $B = (\beta_{jk})$, then $\sigma(AB) = \sum_{i,j} \alpha_{ij}\beta_{ji}$.

We define

$$(2.1) \qquad e(\alpha) = e^{2\pi i t(\alpha)/p}, \qquad t(\alpha) = \alpha + \alpha^p + \cdots + \alpha^{p^{n-1}} \qquad (\alpha \in GF(q)),$$

from which it follows that $e(\alpha + \beta) = e(\alpha)e(\beta)$ and

$$(2.2) \qquad \sum_\beta e(\alpha\beta) = \begin{cases} q & (\alpha = 0) \\ 0 & (\alpha \neq 0), \end{cases}$$

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