POLYNOMIAL SOLUTIONS OF THE CYLINDRICAL WAVE EQUATION

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1. **Introduction.** The conical characteristic value problem for the cylindrical wave equation is the determination on the interior of the characteristic cone

$$t^2 = x^2 + y^2 \qquad (t > 0) \tag{1.1}$$

of that solution $u(x, y, t)$ of the wave equation

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \tag{1.2}$$

assuming prescribed functional values

$$u(x, y, r) = \psi(x, y) \qquad (x = r \cos \theta, y = r \sin \theta) \tag{1.3}$$

on the surface (1.1).

If a solution $u(x, y, t)$ of this problem exists, then the mean-value theorem [1] of Asgeirsson implies that

$$u(0, 0, t_0) = \psi(0, 0) + \frac{t_0^{\frac{1}{2}}}{2\pi} \iint \frac{[x\psi_x + y\psi_y]\, dx\, dy}{r^2 \sqrt{t_0 - 2r}} \qquad 0 \leq 2r \leq t_0 . \tag{1.4}$$

Provided that there exists a Lorentz transformation which leaves invariant both the wave equation and the characteristic cone and takes the point (x_0, y_0, t_0) into the point $(0, 0, \sqrt{t_0^2 - x_0^2 - y_0^2})$, then (1.4) can be used to obtain the value of u at any interior point (x_0, y_0, t_0) of (1.1). This needed Lorentz transformation will now be given explicitly, but first some notational symbolism will be introduced. The notational abbreviations are the following:

$$\begin{aligned} &\Gamma \equiv (t^2 - x^2 - y^2)^{\frac{1}{2}}, \qquad \Phi \equiv (t^2 - y^2)^{\frac{1}{2}}, \qquad \Lambda \equiv (t^2 - x^2)^{\frac{1}{2}}, \\ &\Omega \equiv t - x \cos \theta - y \sin \theta \end{aligned} \tag{1.5}$$

and the symbols Γ_0, Φ_0, Λ_0, Ω_0 which will denote the values of Γ, Φ, Λ, Ω at the point (x_0, y_0, t_0). In terms of this notation the Lorentz transformation is given by

$$\begin{aligned} x &= -\frac{t_0}{\Lambda_0}\xi - \frac{x_0 y_0}{\Lambda_0 \Gamma_0}\eta + \frac{x_0}{\Gamma_0}\zeta \\ y &= \qquad\quad - \frac{1}{\Lambda_0 \Gamma_0}\eta + \frac{y_0}{\Gamma_0}\zeta \\ t &= -\frac{x_0}{\Lambda_0}\xi - \frac{t_0 y_0}{\Lambda_0 \Gamma_0}\eta + \frac{t_0}{\Gamma_0}\zeta. \end{aligned} \tag{1.6}$$

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