EMBEDDING IN ALGEBRAS OF TYPE I

By Harold Widom

Introduction. The theory of AW^*-algebras was instituted by Kaplansky [6] as an attempt to isolate the purely algebraic properties of rings of operators. An AW^*-algebra is a C^*-algebra satisfying one additional axiom which automatically holds in a ring of operators. This axiom guarantees the existence of sufficiently many projections in the algebra. It was shown that many of the important algebraic properties of rings of operators, for instance generalized comparability of projections and the existence of a dimension function for a finite algebra, also hold for AW^*-algebras.

The object of this paper is to show that in a large number of cases (and perhaps always) the technique of the theory of rings of operators can be applied to the study of AW^*-algebras, indeed that these algebras can be considered rings of operators in a certain general sense. To carry this out we would like to find a Hilbert space, or something like a Hilbert space, on which our AW^*-algebra can act like a ring of operators. It does not seem that a Hilbert space is the right object to look for in general. In fact, it is shown in [1, Chapter XI, Corollary 1, Theorem 12] that there exists a commutative AW^*-algebra not isomorphic to a ring of operators on any Hilbert space. A substitute for a Hilbert space, however, does exist. The substitute, an AW^*-module, arose naturally in Kaplansky's study of AW^*-algebras of type I [7]. An AW^*-module is like a Hilbert space except that the field of complex numbers is replaced by an arbitrary commutative AW^*-algebra. It was shown that any AW^*-algebra $\mathfrak{B}$ of type I is isomorphic to the algebra of all bounded operators on some AW^*-module over the center $\mathfrak{z}$ of $\mathfrak{B}$. This result justifies an attempt to determine when an AW^*-albegra has a suitable representation as an algebra of bounded operators on an AW^*-module.

This paper is divided into two parts. In the first we develop the theory of AW^*-modules, the most important point here being that a topology can be found on a commutative AW^*-algebra to replace the ordinary topology on the complex numbers. The second part of the paper is devoted to the study of representations of an AW^*-algebra as an algebra of bounded operators on an AW^*-module.

For the basic theory of AW^*-algebras and AW^*-modules we refer the reader to the series of papers [6]–[8] of Kaplansky.

I. AW^*-Modules

1. **Topologies.** Let $\mathfrak{z}$ be a commutative AW^*-algebra, $\mathfrak{M}$ the maximal ideal space of $\mathfrak{z}$. In its induced weak topology $\mathfrak{M}$ is an extremally disconnected compact Hausdorff space, and $\mathfrak{z}$ is isomorphic to $C(\mathfrak{M})$, the algebra of continuous

Received November 22, 1955.