BOREL SUMMABILITY OF THE CONJUGATE SERIES OF A DERIVED FOURIER SERIES.

By S. D. Sinvhal

1. **Introduction.** Let $f(x)$ be periodic with period 2π and Lebesgue integrable in $(0, 2\pi)$ and let

$$f(x) \sim \frac{1}{2} a_0 + \sum_1^\infty (a_n \cos nx + b_n \sin nx)$$

be its Fourier series. Then the series

$$\sum_1^\infty n(a_n \cos nx + b_n \sin nx) \tag{1.1}$$

is called the series conjugate to the derived Fourier series of $f(x)$. The object of this paper is to give a sufficient condition for the Borel summability of the series (1.1). (If S_n denotes the n-th partial sum of a series $a_0 + a_1 + a_2 + \cdots$, then the series is said to be summable by Borel method (or, in short, summable B) if

$$\operatorname*{Lim}_{r\to\infty} e^{-r} \sum_{n=0}^\infty \frac{r^n S_n}{n!} \tag{1.2}$$

exists and has a finite value S.)

2. **Theorem.** *The series* (1.1) *is summable B to the value of the integral*

$$\lim_{r\to\infty} - \frac{1}{2\pi} \int_{1/r}^{\pi} \frac{\phi(t)}{\sin^2(t/2)}\, dt \tag{2.1}$$

at a point x at which the above integral exists in the Cauchy sense, provided that the conditions

(2.2) (a) $\phi(t) = 0(t)$; *and*
(b) $\phi(t)/\sin(t/2)$ *is of bounded variation to the right of* $t = 0$, *are satisfied.*

Proof. We put

$$\phi(t) \equiv \phi_x(t) = \tfrac{1}{2}[f(x+t) + f(x-t) - 2f(x)].$$

It is well known that S_n, the n-th partial sum of the series (1.1), is

$$S_n = -\frac{1}{2\pi}\int_0^\pi \frac{\phi(t)}{\sin^2(t/2)}\,dt + \frac{1}{2\pi}\int_0^\pi \frac{\phi(t)}{\sin^2(t/2)}\cos nt\,dt + \frac{1}{\pi}\int_0^\pi n\phi(t)\cos nt\,dt + \frac{1}{\pi}\int_0^\pi n\phi(t)\cot\frac{t}{2}\cdot\sin nt\,dt.$$

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