LOCAL CONNECTIVITY OF MAPPING SPACES

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1. In the present note, we shall prove some theorems concerning the local connectivity of some mapping spaces. As a global property, it is well known that the mapping space from an m-sphere into an l-connected space ($l \geq m$), is $(l - m)$-connected. We shall prove in §2 correspondingly, that the mapping space with the compact-open topology, from an m-dimensional finite polyhedron into a locally l-connected space, is locally $(l - m)$-connected. In §3, the similar results for its subspaces, whose elements map some subpolyhedra into corresponding subspaces, are discussed.

2. A space X is said to be *locally l-connected* provided for every point x of X and for every neighborhood U of x in X, there exists an l-connected neighborhood V of x contained in U; namely V satisfies the conditions

$$\pi_i(V) = 0 \qquad (i = 0, 1, \cdots, l) \tag{1}$$

where the vanishing of $\pi_0(V)$ means that V is arcwise connected. We shall also allow l to be ∞.

Let X^Y be a space whose points are continuous mappings from a space Y into a Hausdorff space X, and whose topology is compact-open [1; Chap. X, 19]; that is, the open sets of X^Y are the sets which are generated by the set of the form (C, U), where $C(\subset Y)$ is a compact set, and $U(\subset X)$ is an open set, and $f \in (C, U)$ means that $f(C)$ is contained in U. We shall cite here the following lemma:

Lemma. *If A, B are locally compact, then the space $(X^B)^A$ is homeomorphic with $X^{A\times B}$ and the correspondence*

$$\varphi: (X^B)^A \to X^{A\times B}$$

is given as follows:

$$\varphi(f)(a, b) = (f(a))(b), \qquad f \in (X^B)^A, \qquad a \in A, \qquad b \in B.$$

The proof is found in [1; Chap. X, 21].

The fundamental result of this paragraph is the following:

Theorem 1. *If P is an m-dimensional finite polyhedron and X is locally l-connected ($l \geq m$), then X^P is locally $(l - m)$-connected.*

Proof. For every neighborhood W of f of X^P, there exists an open set $(C_1, U_1) \cap \cdots \cap (C_t, U_t)$ of f in W, where the C_α ($\subset P$) are compact and the U_α ($\subset X$) are open [1; Chap. I, 11].

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