SUBDIRECT SUM REPRESENTATIONS OF PRIME RINGS

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1. **Introduction.** Most of the known results about subdirect sums of rings have to do with conditions under which a given ring has a representation as a subdirect sum of rings of some particular type. However, very little is known about the converse problem in which one has given a set of rings $\{R_\alpha\}$ and seeks to determine whether among the various subdirect sums of these rings there exists one which is isomorphic to a given ring S, or to any ring S of some given type. Naturally, we may call a result of this kind a positive or negative result according as the conclusion reached is that there does, or does not, exist such a subdirect sum. Krull [2] has considered a number of problems of this nature for the case in which the given rings R_α are fields and the ring S is an integral domain (not a field), and has obtained both positive and negative results. For the most part, rather heavy restrictions (e.g., countability) are made on the rings considered, and so the results are of a somewhat special nature. Nevertheless, they seem to be of some interest since they are the first known results of this kind.

An analysis of Krull's proofs reveals that a central feature is the repeated use of the fact that if $\mathfrak{a}$ and $\mathfrak{b}$ are ideals in an integral domain with $\mathfrak{a} \cap \mathfrak{b} = (0)$, then $\mathfrak{a} = (0)$ or $\mathfrak{b} = (0)$. Since an arbitrary prime ring has this property, it is natural to inquire whether his results can be generalized to results about subdirect sums of prime rings. It turns out that all of his results, with one important exception [2; Theorems 10, 10a, 10b], can be so formulated that they carry over to this more general setting.

In this note we shall sufficiently illustrate the methods involved by proving in detail a suitable generalization (Theorem 2) of one of Krull's results. This theorem states that if R is a countable prime ring, with infinite center, which has a proper representation as a subdirect sum of the countable number of prime rings R_i $(i = 1, 2, \cdots)$, then the polynomial ring $R[x]$ also has a proper representation as a subdirect sum of the *same* prime rings R_i $(i = 1, 2, \cdots)$. We also give in Theorem 3 a partial result of this type for the case in which the given prime ring has finite center. Actually, this last theorem has no content in the commutative case, for a field can not have a proper representation as a subdirect sum of any rings.

2. **Definitions and simple properties of prime rings.** A *prime ideal* in a ring R is usually defined to be an ideal $\mathfrak{p}$ with the property that if $\mathfrak{a}$ and $\mathfrak{b}$ are any ideals in R such that $\mathfrak{a}\mathfrak{b} \equiv 0$ $(\mathfrak{p})$, then $\mathfrak{a} \equiv 0$ $(\mathfrak{p})$ or $\mathfrak{b} \equiv 0$ $(\mathfrak{p})$. A *prime ring* is then a ring in which the zero ideal (0) is a prime ideal. It follows that if $\mathfrak{p}$ is an ideal

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