A CLASS OF PRIMITIVE RINGS

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1. **Introduction.** The ring of all linear transformations of a vector space, and the ring of bounded operators on Hilbert space, each have the property that every right (left) annihilating ideal is a principal right (left) ideal generated by an idempotent element. This fact played a key role in the characterization of these rings [15] and [16]. Following a suggestion of Kaplansky, we attempt to identify the class of all primitive rings containing minimal ideals which possesses this property. The primitive ring is first represented as a ring of continuous linear transformations on a pair of dual vector spaces. The annihilating ideals are shown to be just the annihilators of, and retractions on, subspaces closed in Mackey's sense [10]. Every such ideal is generated by an idempotent, if and only if, there exists a continuous idempotent projecting on every closed subspace. The existence of such projections is shown to be equivalent to the existence of a certain type of complement for each closed subspace. For the arbitrary vector space and the Hilbert space, the complementation property reduces respectively to the existence of an ordinary complement for a given subspace, and the existence of orthogonal complements.

These ideas are then used to derive a necessary and sufficient condition for the reflexitivity of a Banach space, in terms of ideal properties of the ring of bounded operators on the space.

2. **Definitions and preliminaries.** To every primitive ring K, containing minimal right ideals, there is associated a pair of dual vector spaces A, B over a division ring F. A and B are linked by an inner product (a, b), a in A, and b in B, which is a non-degenerate bilinear function from $A \times B$ to F. A linear transformation ρ on A is said to have an adjoint ρ^* on B, if ρ^* is a linear transformation on B such that $(a\rho, b) = (a, \rho^* b)$. Those linear transformations which possess adjoints are called continuous. The linear transformation ρ is called finite-valued if the space $A\rho$ is of finite rank. The ring K may be isomorphically represented as a subring $E(A, B)$ of the ring $T(A, B)$ of all continuous linear transformations on (A, B), containing the ring $T_0(A, B)$ of all finite-valued linear transformations on A which have adjoints on B; [3], [5]. We follow the notation of [15]. If P is any subset of K, then $\mathcal{R}(P)$ denotes the totality of x in K such that $Px = 0$. Similarly $\mathcal{L}(P)$ denotes the left annihilator of P. We call such ideals annulets. If S is a subspace of A, then $R(S)$ is the totality of elements ρ in $K = E(A, B)$ such that $S\rho = 0$, and $L(S)$ is the totality of elements α in K such that $A\alpha \leq S$. If P is any subset of K then $N(P)$ is the totality of x in A such that $xP = 0$, and AP is the set of elements ap for a in A

Received February 22, 1954.