FLAT SPACES FOR WHICH THE JORDAN CURVE THEOREM HOLDS TRUE

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The Jordan curve theorem, to the effect that every simple closed curve has only two complementary domains and is the boundary of each of them, was proved [3] by Veblen on the basis of his order axioms I–VIII, XI [4]. While he showed [4] that these axioms are not sufficient to determine that the set of all open curves that he calls straight lines is topologically equivalent to the set of all straight lines of the plane, it is nevertheless true [1] that in every space satisfying these axioms there exists a set of open curves which is topologically equivalent to the set of all straight lines in the plane and accordingly every such space is topologically equivalent to the plane.

In the present paper, the Jordan curve theorem will be established on the basis of Postulates 1-5 listed below. It will also be proved on the basis of a set of postulates consisting of Postulates 1-3, Theorem 2, and the postulate that space is locally arc-wise connected. It will be shown that there exist spaces satisfying all of these postulates and which are not topologically equivalent to the plane, are not metric, and indeed do not even contain a nondegenerate separable continuum.

The undefined terms used are those of point and region. The terms "limit point", "boundary", "mutually separated", and "connected", and the statement that "H separates K and L in M" employed in the statement of these postulates are defined in terms of point and region as in [2].

Arc, simple closed curve, and perfectly compact, are defined as follows.

Definition 1. A point set M is *perfectly compact*, if and only if each monotonic collection of closed subsets of M has a common part.

Definition 2. An *arc* is a *nondegenerate perfectly compact continuum* that does not contain three noncut points. (For the meaning of terms used but not defined in this paper see [2].)

Definition 3. A *simple closed curve* is a *nondegenerate perfectly compact continuum* such that each two points of it separate it.

The letter S is reserved to denote the set of all points.

Postulate 1. *There exists a region, and each region is a nondegenerate connected point set.*

Postulate 2. *If P and Q are two points, there exists a region containing P but not Q.*

Postulate 3. *If two regions contain the point P, then their common part contains a region containing P.*

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