NOTE ON BOUNDS FOR SOME DETERMINANTS

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1. **Introduction.** In a recent paper E. V. Haynsworth [1] introduced several new ideas into the metrical theory of determinants. Some results obtained in this paper can be considerably improved and developed in using some former theorems of mine.

2. **Notations.** We introduce first some notations. Take $n \geq 2$. An $n \times n$ matrix $A = (a_{\mu\nu})$ will be called an *M-matrix* if all diagonal elements are positive, all non-diagonal elements non-positive and if all principal minors of all orders $1, 2, \cdots, n$, of A are positive. For an $n \times n$ matrix A we introduce the expressions

$$(1) \qquad d_\mu(A) = a_{\mu\mu} - \sum_{\substack{\nu=1 \\ \nu\neq\mu}}^{n} | a_{\mu\nu} | \qquad (\mu = 1, \cdots, n).$$

An $n \times n$ matrix will be called an *Hadamard-matrix* if we have

$$(2) \qquad d_\mu(A) > 0 \qquad (\mu = 1, \cdots, n).$$

If in particular an Hadamard-matrix has all its non-diagonal elements non-positive it will be called a *Minkowski-matrix*. As a limiting case we will also consider matrices with non-negative diagonal elements in which the relations (2) hold with $\geq$ sign. If we then have in (2) at least for one μ the equality-sign we will speak of an *improper* Hadamard-matrix or *improper* Minkowski-matrix. Finally, a real $n \times n$ matrix A will be called an *H-matrix* if its diagonal elements are positive and if in replacing in A all non-diagonal elements $a_{\mu\nu}$ by $- | a_{\mu\nu} |$ we obtain an M-matrix. (The class of the M-matrices as described above is identical with the class of matrices of determinants, introduced in [2; 69], as proper M-determinants (eigentliche M-Matrizen). The Hadamard-matrices are matrices of determinants, introduced in [2; 73] as Hadamard-determinants; however, in this paper we require, for simplicity sake, that $a_{\mu\mu}$ are real. Minkowski-matrices are matrices of determinants, introduced in [2; 73] as Minkowski-determinants. Finally, the H-matrices are matrices of the H-determinants, introduced in [2; 70]. We add here however, for simplicity sake, the condition that the diagonal elements are positive.)

In what follows we will denote the maximum and minimum of all non-diagonal elements in the μ-th row of a real matrix A by A_μ and a_μ. The determinant of a matrix A will be denoted by $| A |$.

We will further make use in what follows of a very convenient notation by F. and R. Nevanlinna: we denote generally for a real u by u^+ the expression $\frac{1}{2}(u + | u |)$, i.e., u or 0 according as u is ≥ 0 or ≤ 0.

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