APPROXIMATIONS TO MONOTONE MAPPINGS ON NON-COMPACT TWO DIMENSIONAL MANIFOLDS

By John G. Hocking

Part I. Preliminary Development.

1. **Introduction.** J. W. T. Youngs [12] proved the interesting theorem that *any monotone mapping of a compact 2-manifold onto itself can be approximated arbitrarily closely by homeomorphisms*, a result he later applied to the representation problem for Frechet surfaces [13]. In this paper I generalize Youngs' result to the non-compact 2-manifolds by considering a class of 1-monotone mappings which reduce to simple monotone in the compact case. It will be noted that my approach is quite different from Youngs' and depends on the theory of 1-regular convergence of sets. In Part I of this paper those aspects of 1-regular convergence which I find useful are collected. These include simple extensions of results of G. T. Whyburn on the relation between r-regular convergence and r-monotone mappings.

The second part of this paper deals with 2-manifolds exclusively. In addition to the above-mentioned generalization of Youngs' result, I prove the theorem: *If M is a 2-manifold with boundary B and f is a compact, 1-monotone mapping of M such that $F \mid B$ is monotone, then $f(M)$ is homeomorphic to M.* This extends a result of Roberts and Steenrod [3] who proved the theorem for the case of a compact 2-manifold without boundary, a result later generalized to a non-compact 2-manifold without boundary by V. Martin [2]. Finally, I make use of these results to obtain some applications involving approximation by means of light mappings.

All spaces considered will be assumed to be complete separable metric spaces and will usually be locally compact. Only modulo 2 Vietoris cycles and homologies on compact carriers are employed. All mappings are assumed to be continuous.

Following Whyburn [6] a mapping f of a space X into a space Y will be said to be

(A) *closed*, if the image of every closed set in X is closed in $F(X)$;
(B) *strongly closed*, if the image of every closed set in X is closed in Y;
(C) *compact* if the inverse-image of every compact set in $f(X)$ is compact.

For further details see Whyburn's memoir [6] which is referred to frequently throughout this paper.

Received September 10, 1953. This paper is, in essence, my doctoral dissertation in the University of Michigan. Warm thanks are due to Professor G. S. Young who directed my doctoral program and whose guidance in preparing this paper was invaluable. The research reported here was partially supported by the Office of Naval Research under Contract N8-ONR-71400.