REPRESENTATIVES OF HOMOTOPY CLASSES OF MAPPINGS INTO SPHERES

By Robert L. Plunkett

1. **Introduction.** A mapping (continuous transformation) f of a continuum X onto a space Y is *quasi-monotone* if, for each continuum K in Y with non-empty interior, $f^{-1}(K)$ has only a finite number of components and each of these maps onto K under f. In [4] and [5], G. T. Whyburn proved that every mapping of a locally connected continuum X into the circle S is homotopic to a quasi-monotone mapping of X into S and that every mapping of a cyclic locally connected continuum X into S is homotopic to an interior mapping of X into S.

In this paper spheres of arbitrary dimensions are admitted as the image spaces and theorems of a similar nature are sought. In §2, it is proved that every mapping of a connected polyhedron into a sphere of any dimension $n \geq 1$ is homotopic to a quasi-monotone mapping. In §3, a similar theorem is proved in which the original space is a connected, compact, orientable manifold of dimension m such that $1 \leq m \leq n$. In §4 there is given, for any dimension $n \geq 2$, an example of a locally connected continuum X and a mapping of X onto S_n which is not homotopic to a quasi-monotone mapping.

For use later the following two theorems, due to A. D. Wallace [3], are quoted:

Theorem 1.1. *In order that a mapping f of a locally connected continuum X onto a space Y be quasi-monotone, it is necessary and sufficient that each component of the inverse of any region $R \subset Y$ map onto R under f.*

Theorem 1.2. *In order that a mapping f of a locally connected continuum X onto a space Y be quasi-monotone, it is necessary and sufficient that it admit a factorization $f = f_2 f_1$, where f_1 is monotone and f_2 is light and interior.*

As in the above theorems, X will designate, throughout, a separable metric space. Also, if n is a positive integer, S_n will denote the set of all points $(x_1, x_2, \cdots, x_{n+1})$ of Euclidean $(n+1)$ dimensional space such that $x_1^2 + x_2^2 + \cdots + x_{n+1}^2 = 1$. Since every mapping onto a proper subset of S_n, for any dimension $n \geq 1$, is homotopic to the constant mapping (which is automatically quasi-monotone), it is assumed that all mappings considered are onto mappings.

2. The following lemma characterizes quasi-monotone mappings of locally connected continua onto S_n.

Received October 30, 1953; in revised form, February 24, 1954. The material in this paper constitutes a portion of the author's dissertation at the University of Virginia. The author wishes to express his appreciation to Professors G. T. Whyburn and E. E. Floyd.