GENERALIZED LOCAL CLASS FIELD THEORY
IV. CARDINALITIES

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Introduction. For the previous work on which this depends, and for terminology, see [47], [48], [49], [50], [51]. Let k be a regular local field whose residue class field $[k]$ has characteristic $p \neq 0$ and cardinal number $\aleph \geq \aleph_0$. We are going to prove that our existence theorem [49], [50] cannot be weakened by replacing our condition of analyticity by the weaker condition that all the additive image groups are open. Namely, define a subgroup $a \subset k^{\cdot}$ to be *pseudoanalytic* if it has a conductor and if all the image groups $[a]_{(i)}$, $i \geq 1$, are open in the sense of [47]. We shall show that $k^{\cdot}$ contains $\aleph$ analytic subgroups of finite index and $2^{\aleph}$ pseudoanalytic subgroups of finite index, and shall determine all such groups in some of the simplest special cases.

Although the image groups of a subgroup of $k^{\cdot}$ depend on the choice of a prime element, the rule of dependence is extremely simple and it is easy to verify that the notions "analytic" and "pseudoanalytic" are invariant under change of prime element.

1. **Subgroups of $[k]^+$.** Let $\bar{r}$ ($\cong GF(p)$) be the prime subfield of $[k]$. Then $[k]^+$, considered as vector space over $\bar{r}$, has a Hamel-basis (= independent basis of $[k]^+$ considered as abstract group); since $\aleph$ was an infinite cardinal it follows easily that every such basis must have cardinal $\aleph$. This being so, we see:

Proposition 1. *There are $2^{\aleph}$ $\bar{r}$-linear mappings of $[k]^+$ into $\bar{r}$, $2^{\aleph}$ subgroups of index p^n in $[k]^+$ for each finite $n \geq 1$, and $2^{\aleph}$ subgroups of finite index in $[k]^+$.*

Since the proof follows well-known methods it is omitted. By Corollary 10.3 of [47] and its proof, one sees that there is a one-one correspondence between the open subgroups of $[k]^+$ (in the a.p. topology [47; 4]) and the finite dimensional $\bar{r}$-subspaces of $[k]^+$. Since there are $\aleph$ such subspaces we see:

Proposition 2. *There are $\aleph$ additive polynomials over $[k]$, $\aleph$ open subgroups of index p^n in $[k]^+$ for each finite $n \geq 1$, and $\aleph$ open subgroups of finite index in $[k]^+$.*

2. **Subgroups of index p in $\mathbf{k}^{\cdot}_{(1)}$.**

Proposition 3. *There are $\aleph$ finite algebraic extensions of k. For each $n \geq 1$ there are $\aleph$ cyclic pure ramified extensions of degree p^n over k.*

Proof. Let R be any system of representatives for $[k]^+$ and π any prime element. From Hensel's Lemma it follows that every finite extension of k is obtained by adjoining a root of a polynomial whose coefficients are polynomials in π

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