GENERALIZED LOCAL CLASS FIELD THEORY

III. SECOND FORM OF EXISTENCE THEOREM. STRUCTURE OF ANALYTIC GROUPS.

By G. Whaples

This is a continuation of [42] and [43], preserves their notation, and carries on their numbering. Although Theorem 4 of [43] gives a very simple characterization of norm groups, it is inconvenient for some purposes: 1. The question is left open whether, for every analytic group a, the $u_i(x)$ of Definition 1 can be so chosen that the elements

$$(36) \qquad 1 + u_i(\xi)\pi^i \qquad (\xi \in o,\ i > 0)$$

generate the whole group $a_{(1)}$. 2. The mappings $\xi \rightarrow 1 + u_i(\xi)\pi^i$ of o into $a_{(i)}$ are very disorderly. Homomorphisms would be better. 3. Given a set of $u_i(x)$ satisfying the requirements of Definition 1, it is a difficult problem to determine the index in $k^{\cdot}_{(1)}$ of the group generated by the elements (36). 4. Given two such sets of polynomials it is hard to decide whether the corresponding elements (36) generate the same group or not.

We shall derive a new description of norm groups which is essentially free from these objections. It has the following simple motivation. If k has characteristic p, a nontrivial homomorphism of o into $k^{\cdot}_{(i)}$ is impossible. So we replace o by the ring $\mathfrak{w}$ of Witt-vectors [23] or [36; 119–127] over $[k]$, and use certain mappings and linear operators invented by Artin and Hasse and applied by them and by Šafarevič to explicit reciprocity laws [33], [35], [37], [40]. The works just mentioned were restricted to the case char. $k = 0$ but we shall show that the same methods work also when char. $k = p$ and in fact there is only one place (proof of Lemma 7) where we need to mention the characteristic of k at all. Of course $\mathfrak{w}$ is isomorphic to a subring of o when char. $k = 0$ but not when char. $k = p$. This does not make any difference; the important thing is that both $\mathfrak{w}$ and o always have $[k]$ as homomorphic image. We don't make any use of Witt's formalism: instead of describing $\mathfrak{w}$ as the ring of Witt-vectors we could equally well have described it as the ring of integral elements of the field which is uniquely determined, up to isomorphism, by the properties that it is complete under a discrete valuation, its element p (= sum of p terms each equal to the unit element) is a prime element of its integral domain, and its residue class field is isomorphic to $[k]$. The existence and uniqueness of such a field can be shown without use of Witt-vectors and the simplest proof is due to S. MacLane [39] or [20; Chapter 7]. Since we assume throughout that $[k]$ has no inseparable extensions, Theorem 5 of [39] is all we need.

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