INVERSE LIMIT SPACES

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1. **Introduction.** This paper concerns two diverse aspects of inverse limit spaces, whose only juncture is the basic properties given in §2.

First it is shown that, in the Alexander-Kolmogoroff cohomology theory, the continuity theorem, which is stated in terms of an inverse limit space, is equivalent to the extension and reduction theorems [6]. Spanier [4] has shown that the continuity theorem is valid in this theory. The virtue of the present treatment is not the novelty of the proof but rather its relation to the axioms for a cohomology theory. Eilenberg and Steenrod [1] have shown that a cohomology theory which satisfies the continuity axiom, in addition to seven other axioms, is essentially unique on compact Hausdorff spaces. The structure of the continuity axiom is complicated and it is natural to try to replace it by an axiom, or axioms, simpler in conception. The extension and reduction theorems, assumed as axioms, are one choice and the present proof is so constituted that, allowing modifications for the axiomatic point of view, it demonstrates their equivalence with the continuity axiom. The Alexander-Kolmogoroff theory is chosen as a frame to avoid submerging the ideas in axiomatic considerations.

Next it is assumed that the maps of the inverse system are monotone. It is then shown that the properties of local connectedness and semi-local connectedness are preserved in the passage to inverse limits; also the inverse limit of arcs (simple closed curves) is an arc (a simple closed curve).

2. **Basic properties.** The results given here are all well known and with the exceptions of 2.8 and 2.9 appear in the literature, notably Eilenberg and Steenrod [1], Lefschetz [3], and van Heemert [5]. Properties 2.8 and 2.9 were communicated by A. D. Wallace.

Definition. $(\Lambda, >)$ is a *directed set* if Λ is a set and $>$ is a relation in Λ satisfying:

(i) $\lambda \not> \lambda$

(ii) whenever $\lambda > \mu$ and $\mu > \nu$ then $\lambda > \nu$,

(iii) for λ and μ there is ν such that $\nu > \lambda$ and $\nu > \mu$, where λ, μ, and γ denote elements of Λ. A subset Λ' is said to be *cofinal* in Λ if for each λ in Λ there is λ' in Λ' such that $\lambda' > \lambda$. Henceforth the relation $>$ will be suppressed and Λ will be called a directed set.

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