A BOUNDARY-VALUE PROBLEM FOR ANALYTIC SOLUTIONS OF AN ULTRAHYPERBOLIC EQUATION

By O. G. Owens

Introduction. In this paper there is defined a class $\mathfrak{A}(X, Y^*)$, (2.1), of functions $u(x_1, x_2, y_1^*, y_2^*)$ having the region $\{x_1^2 + x_2^2 \leq 1, y_1^{*^2} + y_2^{*^2} = 1\}$ as domain and, by interchanging the variables, a class $\mathfrak{B}(X^*, Y)$ of functions $u(x_1^*, x_2^*, y_1, y_2)$ having the region $\{x_1^{*^2} + x_2^{*^2} = 1, y_1^2 + y_2^2 \leq 1\}$ as domain. Any function $u(x_1, x_2, y_1^*, y_2^*)$ of class $\mathfrak{A}(X, Y^*)$ or function $u(x_1^*, x_2^*, y_1, y_2)$ of class $\mathfrak{B}(X^*, Y)$ will be called an analytic admissible boundary function. We define one other class $\mathfrak{F}(X, Y)$, (3.1), of functions $u(x_1, x_2, y_1, y_2)$ having $\{x_1^2 + x_2^2 \leq 1, y_1^2 + y_2^2 \leq 1\}$ for domain. Functions of this class $\mathfrak{F}(X, Y)$ are called analytic admissible functions.

Our principal result, (4.6), asserts that there exists an analytic admissible $u(x_1, x_2, y_1, y_2)$ which is a solution of the ultrahyperbolic partial differential equation

$$\text{(E)} \qquad \frac{\partial^2 u}{\partial x_1^2} + \frac{\partial^2 u}{\partial x_2^2} = \frac{\partial^2 u}{\partial y_1^2} + \frac{\partial^2 u}{\partial y_2^2}$$

on the region $\{x_1^2 + x_2^2 \leq 1, y_1^2 + y_2^2 \leq 1\}$ and which coincides with the arbitrarily given analytic admissible boundary function

$$\text{(B.V)} \qquad u(x_1^*, x_2^*, y_1, y_2) = f(x_1^*, x_2^*, y_1, y_2)$$

on $\{x_1^{*^2} + x_2^{*^2} = 1, y_1^2 + y_2^2 \leq 1\}$, provided $f(x_1^*, x_2^*, y_1, y_2)$ belongs to a certain subclass, (4.6), of $\mathfrak{B}(X^*, Y)$. Moreover, the solution $u(x_1, x_2, y_1, y_2)$ is unique, (4.2), with respect to analytic admissible functions.

1. **Notation.** $x = (x_1, x_2)$ and $p = (p_1, p_2)$ will denote points of the 2-dimensional Cartesian X-space and $y = (y_1, y_2)$ and $q = (q_1, q_2)$ points of the 2-dimensional Cartesian Y-space. Euclidean distances in these two spaces will be indicated by symbols such as:

$$\text{(1.1)} \qquad \begin{aligned} |px| &= [(p_1 - x_1)^2 + (p_2 - x_2)^2]^{\frac{1}{2}}, \qquad & |x| &= [x_1^2 + x_2^2]^{\frac{1}{2}} \\ |qy| &= [(q_1 - y_1)^2 + (q_2 - y_2)^2]^{\frac{1}{2}}, \qquad & |y| &= [y_1^2 + y_2^2]^{\frac{1}{2}}, \end{aligned}$$

and we shall agree that $x^2 = x_1^2 + x_2^2$, $y^2 = y_1^2 + y_2^2$, *etc.* Starring a letter will mean that the corresponding point is at unit distance from the origin, for example, x^* indicates a point of X-space with $|x^*| = 1$. The unit circle $|x| \leq 1$ ($|y| \leq 1$) will be denoted by $C(X)$ ($C(Y)$) and its boundary $|x^*| = 1$ ($|y^*| = 1$) by $C^*(X)(C^*(Y))$. The Green's function for the 2-dimensional Laplacian on $C(X)(C(Y))$ will be denoted by $K(p, x)(K(q, y))$. The usual

Received February 18, 1953.