THEOREMS ON FIBRE SPACES

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In 1950, S. T. Hu [1; §1] gave a definition of fibre spaces which included the fibre bundles. The original purpose of the present work was to further elucidate the relation between fibre spaces so defined and fibre bundles. Theorem 1 gives a condition sufficient to insure that such a fibre space be an E-F bundle, and thus in certain cases a fibre bundle. As an application of this result, we show that every fibre space satisfying certain connectedness conditions is an E-F bundle (Theorem 4).

Steenrod [4; §11] proved the first and second covering homotopy theorems for fibre bundles under the restriction that the domain of the function being deformed be a C_σ-space (*i.e.*, normal, locally compact, and such that every open covering has a countable subcovering). The arguments may obviously be generalized to fibre spaces [2; §26]. In §2 we obtain the covering homotopy theorems for the particular case of a fibre space with totally disconnected fibres with no restrictions on the domain of the function being deformed. In that section there are also certain related results, including a corollary to the effect that certain fibre spaces always admit cross sections.

Covering homotopy theorems for fibre bundles with totally disconnected groups have recently been proved by H. Miyazaki [3] under the assumption that the domain of the function being deformed be connected and locally arcwise connected. The relation between these and the present results is of interest: both seem to depend essentially on the lifting of families of paths in a manner sufficiently smooth to insure continuity of the resulting covering function.

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1. **Fibre spaces and fibre bundles.** A triple $\{X, B, \pi\}$ is called a *fibre structure* if X and B are topological spaces, $\pi: X \to B$ is a continuous function onto, and there is a pair $\{\Omega, \phi\}$ such that

1) Ω is an open covering for B,
2) ϕ is a family of functions, indexed by Ω, such that if $U \in \Omega$ then $\phi_U : U \times \pi^{-1}(U) \to \pi^{-1}(U)$ is continuous,
3) if $U \in \Omega$, $b \in U$, and $x \in \pi^{-1}(U)$, then

$$\pi\phi_U(b, x) = b \qquad \text{and} \qquad \phi_U(\pi(x), x) = x. \tag{1}$$

We also say that X is a *fibre space* over B relative to π; B is the *base space*, and π is the *projection*. The pair $\{\Omega, \phi\}$ is called a *slicing system* for $\{X, B, \pi\}$. If $b \in B$, then $\pi^{-1}(b)$ is called the *fibre* over b.

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