BASIC SETS OF POLYNOMIALS AND THEIR RECIPROCAL, PRODUCT AND QUOTIENT SETS

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Introduction. For the general terminology used in this paper the reader is referred to J. M. Whittaker's books [6], [7].

In §1, I introduce formulas for finding the order and the type of some classes of basic sets. The formulas due to Whittaker involve maxima of the basic polynomials on fixed circles $| z | = R$. My treatment replaces Whittaker's formulas by formulas involving only a single term of each polynomial, and can be therefore regarded as simpler in some applications. For the class of basic sets considered by me, I also obtain formulas for the *upper bound* for the order of product sets. These results are obtained quite generally without any restrictions on the coefficients of the constituent sets. I also introduce the concept of *quotient sets,* and obtain similar results for them. The formulas introduced here easily yield previous results given by M. Nassif [5], Mursi and Makar [4], and M. T. Eweida [1], [2].

In §2, I apply the same principle of considering a single maximum term and obtain new formulas to express the *range where certain basic sets are effective,* and the results are also extended to *reciprocal, product and quotient sets.* In particular, the new formulas do not require the actual calculation of the product and quotient sets. I also obtain results on the effectiveness of simple sets with bounded coefficients.

Definition 1. *If* $p_n(z) = \sum_i p_{ni}z^i$ *and* $\mathbf{p}_n(z) = \sum_i \pi_{ni}z^i$, *then* $\{\mathbf{p}_n(z)\}$ *is called the reciprocal set of* $\{p_n(z)\}$.

Definition 2. *If* $\{p_n(z)\}$, $\{q_n(z)\}$ *are basic sets of polynomials such that*

$$z^n = \sum_i \pi_{ni}p_i(z) = \sum_i \lambda_{ni}q_i(z)$$

then the product set

$$\{u_n(z)\} \equiv \{p_n(z)\}\{q_n(z)\}$$

in this order is defined by $u_{ij} = \sum_h p_{ih}q_{hj}$, *and for the associated operators we have the relation* $\mu_{ij} = \sum_h \lambda_{ih}\pi_{hj}$, *where* $z^n = \sum_i \mu_{ni}u_i(z)$.

Definition 3. *If* $\{u_n(z)\}\{q_n(z)\} \equiv \{p_n(z)\}$, *then* $\{u_n(z)\} \equiv \{p_n(z)\}\{\mathbf{q}_n(z)\}$, *and we call* $\{u_n(z)\}$ *the quotient set of the two sets* $\{p_n(z)\}$, $\{q_n(z)\}$ *in this order; we have* $\mu_{ij} = \sum_h q_{ih}\pi_{hj}$ *and* $u_{ij} = \sum_h p_{ih}\lambda_{hj}$.

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