FINITE METABELIAN GROUPS AND PLANES OF $\sum_{14}$

By Leland L. Scott

1. **Introduction.** We are interested in finite metabelian groups G whose elements, except the identity, are all of order p. The metabelian groups are those groups whose commutator subgroup K is contained in the central C; hence, G/C is abelian and of type 1, 1, $\cdots$. We shall assume that the commutator subgroup K and the central C coincide. If not, C is the direct product of K and an abelian group C', and G is the direct product of C' and a metabelian group G' which possesses all the interesting properties of G [1], [2]. H. R. Brahana has classified groups G which are subgroups of the holomorph of one of their abelian subgroups; thus, we assume G does not have this property [1].

In general, G has the form $G = \{C, U_1, \cdots, U_k\}$ where C is of order p^c with $k - 1 \leq c \leq k(k-1)/2$ and G is of order p^{c+k}. Such groups have been classified for $k \leq 4$ [2], and for $k = 5$ [3]. For $k = 6$, the classification has been completed for $c = 15$, 14, and 13 [5]. This paper deals with those groups for which $k = 6$ and $c = 12$.

The group $\mathfrak{G}$ for which $c = k(k-1)/2$ has been referred to by Brahana as the Master Group [2]. $\mathfrak{G}$ is completely determined by k since a simple isomorphism of two groups with the same k is exhibited by letting generators correspond and by letting commutators of corresponding pairs of generators corresond. For $k = 6$, $\mathfrak{G}$ is generated by $U_1, U_2, U_3, U_4, U_5, U_6$, all of order p, and we have the additional defining relations for the commutators $C_{ij} = U_i^{-1}U_j^{-1}U_iU_j$ $(i < j; i = 1, \cdots, 5; j = 2, \cdots, 6)$.

Other groups for $k = 6$ are obtained by imposing additional conditions on the commutators. This results in equating a certain subgroup of the commutator subgroup to identity and the resulting group G may be thought of as the quotient-group of $\mathfrak{G}$ with respect to the subgroup of C which was equated to identity. In our case, we will be considering quotient-groups of the master group $\mathfrak{G}$ determined by subgroups of order p^3.

2. **Geometric statement of the problem.** The problem of groups is reduced to a problem of geometry by the following considerations:

The general element of G is $g = c\prod_1^6 U_i^{x_i}$ where c is in C. To a point $(x_1x_2x_3x_4x_5x_6)$ of the projective five-space S_5, there corresponds the subgroup, $\{C, \prod_1^6 U_i^{x_i}\}$, of G.

Similarly, the general element of C is $c = \prod_{i<j} C_{ij}^{a_{ij}}$, $(i = 1, \cdots, 5; j = 2, \cdots, 6)$, and to the cyclic subgroup of order p generated by c, there corresponds the point $(a_{12}a_{13}a_{14}a_{15}a_{16}/a_{23}a_{24}a_{25}a_{26}/a_{34}a_{35}a_{36}/a_{45}a_{46}/a_{56})$ of the projective fourteen-space $\sum_{14}$.

Received June 17, 1952; in revised form, December 31, 1952.