

AN EXTENSION OF RESULTS IN THE UNIQUENESS THEORY OF DOUBLE TRIGONOMETRIC SERIES

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1. **Introduction.** It is the purpose of this paper to extend the results obtained by Cheng [2] in the uniqueness theory of double trigonometric series summed in a circular manner.

Whenever convenient, vectoral notation will be used and will be signified by capital letters thus:

$$M = (m, n), \quad X = (x, y), \quad MX = mx + ny, \quad |M| = (m^2 + n^2)^{\frac{1}{2}}.$$

A double trigonometric series

$$(1) \quad \sum_M a_M e^{iMX},$$

where the a_M are arbitrary complex numbers which are $o(1)$, that is $a_M \rightarrow 0$ as $|M| \rightarrow \infty$, will be said to converge circularly at the point X to $L(X)$ if the circular partial sums of rank R

$$(2) \quad S_R(X) = \sum_{|M| \leq R} a_M e^{iMX}$$

converge to the finite value $L(X)$. The series will be $(C, 1)$ circularly summable to $L(X)$ if the $(C, 1)$ circular mean of rank R ,

$$\sigma_R(X) = \frac{2}{R^2} \int_0^R S_u(X) u \, du \equiv \frac{S_R^{(1)}(X)}{R^2},$$

converges to the finite value $L(X)$. The series will be of type (U) if

$$\sum_{1 \leq |M| \leq R} \frac{a_M e^{iMX}}{|M|^2}$$

converges uniformly in X .

In this paper the fundamental square

$$\{(x, y); 0 \leq x \leq 2\pi, 0 \leq y \leq 2\pi\}$$

will be designated by Ω .

All sets Z of capacity zero will be considered closed sets.

2. **Statement of main results.** In his work on uniqueness theory Cheng [2] obtained results which are tantamount to the following:

(a) If the double trigonometric series (1.1) is of type (U) and is $(C, 1)$ circularly summable to zero everywhere, then the series vanishes identically.

(b) If the double trigonometric series (1.1) is of type (U) and is $(C, 1)$ cir-

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