METRIZATION OF FINITE DIMENSIONAL GROUPS

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1. **Introduction.** It seems likely that any locally compact, connected, finite dimensional topological group is separable metric. In the following we show that this is the case if any one of several additional hypotheses is made; namely, in case the group is also compact (§2), or locally connected (§3), or an (L)-group (§4), or 1-dimensional (§5). In the case of compact groups this fills a gap in the structure theory, and similar remarks also apply to the other cases.

One must, of course, be explicit concerning the definition of dimension used, since the groups in question are not presupposed to be separable metric. We adopt the homology dimension. More explicitly, if X is a compact Hausdorff space, let $H_n(X)$ be the n-th Čech homology group of X with reals modulo 1 as coefficient group. Then, by definition, dim $(X) \leq n$ if and only if, for every closed subset C of X, the natural homomorphism of $H_n(C)$ into $H_n(X)$ is an isomorphism into (compare [6;151, Theorem VIII 3′]). If X is a locally compact Hausdorff space, dim (X) is defined to be the supremum of the set of dimensions of compact subsets of X. Recently this dimension function has been studied by Haskell Cohen [4], who used an equivalent cohomological version (see also [1]). We use a few of his results, to wit: (1) if X is a locally compact Hausdorff space, and C is a closed subset, then dim $(C) \leq$ dim (X); and (2) if E^n is an n-cell (open or closed), dim $(E^n) = n$.

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2. **Compact groups.** In this section we prove that a compact, connected, finite dimensional group is separable metric. The proof depends on the fact that any such group is isomorphic to the inverse limit of compact, connected Lie groups [14; 88]. We proceed by first proving the theorem for limits of Lie groups of some fixed dimension, and then showing that this is actually the general case.

LEMMA 2.1. *If G is the inverse limit of compact, connected, n-dimensional Lie groups, then G is isomorphic to $(C \times S)/D$, where C is the limit of r dimensional toruses, S is the limit of compact, connected, semi-simple $n - r$ dimensional Lie groups, and D is a totally disconnected, closed, central subgroup.*

Proof. This proof depends, as does that of Lemma 2.8, on the theory of the structure of compact, connected Lie groups. Let $G = \lim \{G_\alpha;f_{\alpha\beta}\}$, let C_α be the identity component of the center of G_α, and let S_α be the semi-simple

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