A NOTE ON MEASURES IN BOOLEAN ALGEBRAS

By Edwin Hewitt

1. **Introductory remarks.** Kappos [2], [3] has considered the notion of a countably additive measure on a Boolean algebra, and a correction to Kappos [2] has been published recently by Sikorsky [4]. The definition of countable additivity given by both of these authors is essentially the following. Let $\mathfrak{A}$ be an abstract Boolean algebra with partial ordering $\leq$ and with Boolean operations $+$, $\cdot$, and $'$. A real-valued function ϕ defined on $\mathfrak{A}$ is a finitely additive measure if

$$\sup_{a \in \mathfrak{A}} | \phi(a) | < +\infty \tag{1.1}$$

and

$$\phi(a + b) = \phi(a) + \phi(b) \text{ for all } a, b \in \mathfrak{A} \text{ such that } ab = 0. \tag{1.2}$$

If

(1.3) $\lim_{n\to\infty} \phi(a_n) = 0$ for all decreasing sequences $\{a_n\}_{n=1}^{\infty} \subset \mathfrak{A}$ such that

$$\prod_{n=1}^{\infty} a_n = 0,$$

then ϕ is said to be countably additive. It is simple to verify that ϕ is countably additive if and only if

$$\phi\left(\sum_{n=1}^{\infty} b_n\right) = \sum_{n=1}^{\infty} \phi(b_n) \qquad \text{for all} \qquad \{b_n\}_{n=1}^{\infty} \subset \mathfrak{A} \tag{1.4}$$

such that

$$\sum_{n=1}^{\infty} b_n \in \mathfrak{A} \qquad \text{and} \qquad b_n b_m = 0 \qquad \text{if} \qquad n \neq m.$$

Although the operations $+$ and $\cdot$ are defined *a priori* only for finite sets of elements in $\mathfrak{A}$, the inclusion relation $\leq$ in $\mathfrak{A}$ gives us an unequivocal interpretation for the symbols $\sum$ and $\prod$ in terms of least upper bounds and greatest lower bounds, respectively, whenever these bounds exist.

For algebras of sets, on the other hand, we have a definition of countable additivity which is quite different from (1.3). Let $\mathfrak{a}$ be an algebra of subsets of a set X. A real-valued function ϕ defined for all $A \in \mathfrak{a}$ is, for our purposes, a finitely additive measure if

$$\sup_{A \in \mathfrak{a}} | \phi(A) | < +\infty; \tag{1.5}$$

$$\phi(A \cup B) = \phi(A) + \phi(B) \text{ for all } A, B \in \mathfrak{a} \text{ such that } A \cap B = 0. \tag{1.6}$$

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