1. Introduction. The Dedekind sum $s(h, k)$ is defined by means of \((\frac{h}{k})\) \((\frac{hr}{k})\) (mod k)
where \((x) = x - [x] - \frac{1}{2}\) when \(x\) is not an integer, \((x) = 0\) when \(x\) is an integer. The most interesting property of the sum is the reciprocity formula
\[
12hk\{s(h, k) + s(k, h)\} = h^2 + 3hk + k^2 + 1.
\]
Elsewhere, the writer has used the representation
\[
s(h, k) = \frac{1}{4k} + \frac{1}{k} \sum_{r \neq 1} \frac{1}{(\zeta - 1)(\zeta^k - 1)},
\]
where \(\zeta\) runs through the \(k\)-th roots of unity, to give a simple proof of (1.2) and indeed of Apostol’s generalization [1].

The object of the present paper is to discuss some analogs of (1.1) in the field \(GF(q, x)\). The straightforward analog of (1.1) is of little interest in the present case. Instead we consider an analog suggested by the representation (1.3). We are thus led to a set of reciprocity theorems similar to (1.2) as well as to a number of related formulas that may be of interest in themselves. The discussion makes use of certain functions previously defined by the writer [2], [3]; the relevant properties are reproduced below.

2. Notations and preliminaries. By \(GF(q, x)\) we shall understand as usual the field of rational functions of the indeterminate \(x\) with coefficients in \(GF(q)\). By \(\Phi = GF\{q, x\}\) is meant the field consisting of the numbers
\[
\alpha = \sum_{r = 0}^{m} c_r x^r \quad (c_r \in GF(q)),
\]
where \(m\) is an arbitrary integer (positive, negative or 0); if \(c_m \neq 0\) we define \(\deg \alpha = m\), \(|\alpha| = q^m\), so that \(|\alpha|\) is a valuation for \(\Phi\). For our purpose we shall also require the extension \(\Phi' = \Phi(x)\), where \(x^{q^m-1} = x\).

We next define [2], [3]
\[
\psi(t) = \sum_{r = 0}^{m} (-1)^r \frac{t^r}{F_r} \quad (t \in \Phi'),
\]
where
\[
F_r = \prod_{s = 0}^{r-1} (x^s - x^r), \quad F_0 = 1;
\]
Received August 9, 1952.