SETS OF CONVERGENCE OF TAYLOR SERIES. II.

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1. **Summary.** In an earlier paper [7] the authors have shown that if M is a set of type F_σ on the unit circle C, there exists a Taylor series which has M as its set of convergence, *i.e.*, which converges on M and diverges on $C - M$. The main purpose of the present paper is to exhibit Taylor series whose sets of convergence are not of type F_σ. Essentially two methods will be used.

A rudimentary version of the first method, which is geometric in spirit, was introduced by P. Erdös and the present authors [3] in the construction of schlicht functions whose Taylor series converge uniformly but not absolutely on C. Given any denumerable set M on C, this method permits the construction of a schlicht function which is regular on $C - \overline{M}$ and whose Taylor series has M as its set of divergence. The method can also be used to construct a bounded schlicht function whose Taylor series has a set of divergence which has locally the power of the continuum.

The second method is based more directly on numerical considerations than the first. A modification of the method developed in [7], it gives no control over the univalence of the functions that are constructed, but permits a certain amount of manipulation of continuity properties.

In §8, the geometrical method is used to exhibit a schlicht function whose Taylor series converges everywhere on C and whose convergence is locally non-uniform on C. The last section is devoted to the characterization of sets of boundedness of Taylor series converging everywhere on C.

2. **On schlicht regions with spiral appendages.** Let the region R consist of the unit disc in the w-plane, together with a spiral extension that wraps itself around the circle $| w - 2 | = 1/2$, and let $w = f(z)$ map the unit disc in the z-plane conformally upon the region R. Then there exists on the unit circle $| z | = 1$ a point z_1 whose radius vector is mapped into a spiral curve that also wraps itself around the circle $| w - 2 | = 1/2$. The Taylor series of $f(z)$ diverges at $z = z_1$ and converges everywhere else on C. For at $z = z_1$, $f(z)$ does not have radial continuity; the convergence at other points on C follows from Fejér's theorem (see Fejér [5] or Landau [8; 65–67]).

Next, let R be modified by the addition of other (and smaller) spiral appendages (see Figure 1); nothing prevents the existence of small spiral appendages that arise from the loops of greater appendages, and it becomes clear how a schlicht function $f(z)$ can be constructed so that its Taylor series diverges on a denumerable set of points which is dense on C, and converges everywhere else

Received July 23, 1952.