CONCERNING A PROBLEM OF SOUSLIN'S

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In the first issue of Fundamenta Mathematicae, Souslin [1] raised the question of the existence of a connected, linearly ordered space which is not separable and does not contain uncountably many mutually exclusive segments. Let a space have property X if and only if it is not separable and does not contain uncountably many mutually exclusive domains. It is easily shown [3] that if there exists a linearly ordered space having property X, there also exists a connected linearly ordered space having property X.

The first three parts of R. L. Moore's Axiom 1 of [2] state that:

There exists a sequence G_1 , G_2 , G_3 , $\cdots$ such that (1) *for each n, G_n is a collection of regions covering S,* (2) *for each n, G_{n+1} is a subcollection of G_n ,* (3) *if R is any region whatsoever, X is a point of R and Y is a point of R either identical with X or not, then there exists a natural number m such that if g is any region belonging to the collection G_m and containing X then $\bar{g}$ is a subset of $(R - Y) + X$.*

Call this Axiom 1_3 .

It is easily shown [4; 628, Theorem 12] that no linear space having property X satisfies Axiom 1_3 . It has been shown in [5] that there exists a locally connected space satisfying Axiom 1_3 and having property X, but that there does not exist a locally connected space satisfying Axiom 1_3 and having property X such that each two points can be separated by a finite point set. The theorems of this paper will prove that a necessary and sufficient condition that there exist a linear space having property X is that there exist a locally connected space satisfying Axiom 1_3 and having property X such that each two points can be separated by either a countable or a separable point set.

Theorem 1. *If there exists a locally connected space having property X such that each two points can be separated by a separable point set, then there exists a linear space having property X.*

Proof. Suppose there is a locally connected space A having property X such that each two points can be separated by a separable point set. There is an uncountable well-ordered sequence b such that (1) each term of b is a countable collection of mutually exclusive connected domains in A, (2) the first term of b is the set of all nondegenerate components of A, (3) if x is an element of a term X of b, there is a separable closed point set s_x which separates x; then y is a domain of the term of b following X if and only if for some element z of X, y is a nondegenerate component of $z - s_z$, and (4) if X_1 , X_2 , X_3 , $\cdots$ is a subsequence of b, then x belongs to the first term of b following the terms of this subsequent

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