LAPLACE-TRANSFORMS. (XI)
THE SINGULARITIES OF LAPLACE-TRANSFORMS. (III)

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1. Introduction. Let us put

$$F(s) = \int_0^\infty \exp(-sx)\, d\alpha(x) \qquad (s = \sigma + it), \tag{1.1}$$

where $\alpha(x)$ is of bounded variation in any finite interval $0 \leq x \leq X$, X being an arbitrary positive constant. In this note we shall study the relation between singularities and boundary values of (1.1).

We shall begin with

DEFINITION. If (1.1) is simply convergent for $\sigma > 0$, we call the point s_0 on $\sigma = 0$ *Picard's point*, if (1.1) assumes every value, except perhaps two (∞ included), infinitely many times, in the half-circle: $|s - s_0| < \epsilon$, $\sigma > 0$, where ϵ is an arbitrary positive constant.

The main theorem is the following.

MAIN THEOREM. *Let* (1.1) *be simply convergent for* $\sigma > 0$. *Then* $s = 0$ *is Picard's point of* (1.1), *provided that*

(a) $\overline{\lim\limits_{m\to\infty}} \, | O_m |^{1/m} \geq 1$,

(b) $\overline{\lim\limits_{m\to\infty}} \, 1/\log m \cdot \log^+ \log^+ | O_m | > \frac{1}{2} \qquad (\log^+ x = \max\{0, \log x\})$,

where

$$O_m = (e/m)^m \int_{m(1-\omega)}^{m(1+\omega)} x^m \exp(-x)\, d\alpha(x) \qquad (0 < \omega < 1).$$

Remark. By Lemma 1, (a) is the necessary-sufficient condition for $s = 0$ to be singular for (1.1).

2. Lemmas. To prove the main theorem, we need some lemmas.

LEMMA 1. (A. Ostrowski [4], [1; 12–16]): *Let* (1.1) *be simply convergent for* $\sigma > 0$. *The necessary-sufficient condition for* $s = 0$ *to be singular for* (1.1) *is that*

$$\overline{\lim_{m\to\infty}} \, | O_m(\sigma; \omega_m, \omega_m') |^{1/m} \geq 1,$$

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