EXCEPTIONAL VALUES OF ENTIRE AND MEROMORPHIC FUNCTIONS

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1. **Introduction.** This paper is a continuation of my paper [5] and we follow the same notation.

Let $f(z)$ be an entire function of finite order ρ and α be any (finite) complex number. Then

$$f(z) - \alpha = z^n P(z,\alpha) \exp \{Q(z,\alpha)\} \tag{1}$$

where

$$Q(z,\alpha) = az^{q(\alpha)} + \cdots, \qquad a = Te^{i\beta} \neq 0,$$

is a polynomial of degree $q(\alpha)$ and $P(z, \alpha)$ is a canonical product (c.p.) of order $\rho_1(\alpha)$ and genus $p(\alpha)$. Let E denote the set of positive non-decreasing functions $\phi(x)$ such that

$$\int_A^\infty \frac{dx}{x\phi(x)}$$

is convergent. (The condition that $\phi(x)$ be non-decreasing in Shah's theorem [4; 23] is not necessary; see Boas [1]). If for some $\phi \subset E$

$$\liminf_{r\to\infty} \frac{T(r)}{n(r,\alpha)\phi(r)} > 0, \tag{2}$$

we say α is an exceptional value (e.v.) E for $f(z)$. If α is an e.v. E then the order ρ is an integer and we have either (i) $\rho_1(\alpha) < \rho = q(\alpha)$ or (ii) $q(\alpha) = \rho = \rho_1(\alpha); p(\alpha) = \rho - 1$. Conversely if (i) holds then α is an e.v. E. In § 6 we show by means of an example that if (ii) holds then α may or may not be an e.v. E. We also prove

Theorem 1. *Let $f(z)$ be an entire function of finite order ρ and let α be* e.v. E *for $f(z)$. Then given an arbitrarily small $\delta > 0$, there exist ρ sectors with center at the origin defined by*

$$\left| \arg z - \left(2\nu + 1 - \frac{\beta}{\pi}\right)\frac{\pi}{\rho} \right| \leq \frac{\pi}{2\rho} - \delta; \qquad \nu = 0, 1, \cdots \rho - 1, \tag{3}$$

in which $f(z) \to \alpha$ uniformly as $z \to \infty$. The number of finite asymptotic values of $f(z)$ is ρ.

This result cannot be extended to meromorphic functions. In fact we have

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