GENERALIZED CONVEX SETS IN THE PLANE

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1. **Introduction.** This paper is an outgrowth of the consideration of generalized convex functions as presented by E. F. Beckenbach [1] and E. F. Beckenbach and R. H. Bing [2]. Later M. M. Peixoto [5] combined Beckenbach's generalization with the convexity of a plane set of points. The present paper deals with the convexity of a plane set of points with respect to a more general family of curves $\{C\}$ than that used by Peixoto. Our investigations are almost entirely nonmetric in character, in distinction from those of H. Busemann [3], pp. 84–94, relative to the establishment of a metric for the family $\{C\}$. J. W. Green and W. Gustin [4] have considered an interesting generalization of convexity which will be discussed at the end of this paper.

2. **Notation and definition of the family** $\{C\}$. We shall consider a family of curves $\{C\}$ in the complex plane, or on the Riemann sphere, which satisfy the following conditions:

(1) Each $C \in \{C\}$ is a closed Jordan curve which passes through the point ω at infinity, or through the north pole of the Riemann sphere.

(2) There is a unique member of the family $\{C\}$ which passes through two finite points in the complex plane.

Herein lower case letters will always denote finite points in the complex plane. If p_i, p_j are any two points on $C_k \in \{C\}$, then briefly $A(C_k\,;\, p_i\,,\, p_j)$ will denote the open arc on C_k from p_i to p_j which does not contain the point ω. The notation $A(C_k\,;\, p_i\,,\, \omega)$ will denote either of the open arcs of C_k from p_i to ω, and a bar over the A will denote the closure of this open arc. We shall also consider the members of $\{C\}$ to be point sets.

We shall say that a curve $C \in \{C\}$ is a bounding curve of a point set E provided all points of E lie in one of the two open regions determined by C, and that C is a supporting curve of E provided E lies in one of the two closed regions determined by C and at least one point of E lies on C. An open two-dimensional sphere with center at p_0 and finite radius will be denoted by $S(p_0\,,\, \delta)$ or at times briefly by S.

3. **The family** $\{C\}$. The following lemma follows from the theorem of Jordan.

LEMMA 1. *Let C_1 and C_2 be two distinct members of the family $\{C\}$ such that C_1 and C_2 pass thru the point p_0. Then C_2 does not lie entirely in the closure of one of the regions of the plane determined by C_1.*

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