GENERALIZED LOCAL CLASS FIELD THEORY

I. RECIPROCITY LAW

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Introduction. We present here a proof of the reciprocity law of the generalized local class field theory of Moriya, Nakayama, and Schilling by a new method which not only replaces simple algebras by cocycles but, in our opinion, simplifies the algebraic part of the proof and clarifies the connection between the cyclic 2-cocycles and the reciprocity law.

This generalized class field theory consists of proofs of the reciprocity law (by which we mean the theory of the norm residue symbol complete with norm transfer and isomorphism transfer theorems), limitation theorem, and the (necessarily weakened [15]) existence theorem, for all fields k satisfying the following axioms (residue class field of k is the ring of all elements of value at most one modulo its ideal of elements of value less than one):

Axiom 0. *k is complete with respect to a non-Archimedean discrete rank one valuation.*

Axiom 1. *The residue class field of k has no inseparable extensions.*

Axiom 2. *The residue class field of k has (in an algebraic closure) exactly one extension of degree n for each positive integer n.*

A complete exposition can be found either in the papers [11], [12], [13], [14], [16], [17], [18] by Moriya and Nakayama, or in the book [20] by Schilling. Many of the results had been published much earlier in various papers by each of these three authors; for these early references see [20].

We shall call fields which satisfy Axiom 0 *local fields* and those which satisfy Axioms 0, 1, and 2 *regular fields*. Local class field theory had previously been known only for those fields—which we shall call *strongly regular*—which besides 0, 1, 2 satisfy the condition that their residue class field be a Galois (i.e. strictly finite) field. Every strongly regular field is either a field of formal power series or a p-adic number field [3; 200], [20; 216], [23]. There are many examples of regular fields which are not strongly regular: closures of infinite algebraic extensions of p-adic number fields [15], or else formal power series fields over a suitable infinite extension of a Galois field or over a field which is itself a formal power series field [19].

Hochschild's methods [9] are not enough to eliminate the simple algebras from the existing proofs in this general case because these proofs use simple [algebras to prove a certain arithmetic result, namely, the theorem that for cyclic extensions the norm index equals the degree—or its equivalent—and

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