# NON-OSCILLATORY DIFFERENTIAL EQUATIONS

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1. **Introduction.** In this paper we shall be concerned with the differential equation

$$(p(x)y')' + f(x)y = 0, \tag{1}$$

where $p(x)$ and $f(x)$ are real-valued functions defined for all non-negative $x$, $p(x)$ being positive, and $p^{-1}(x)$ and $f(x)$ belong to $L(0, R)$ for every large $R$. A solution of (1) is a function $y(x)$ having the properties that it is absolutely continuous and satisfies the equation (1) for almost all $x$ if $p(x)y'(x)$ is replaced by an absolutely continuous function which is equal to $p(x)y'(x)$ almost everywhere. In the sequel only those solutions which are real-valued and are distinct from the trivial solution ($\equiv 0$) shall be considered.

On the positive $x$-axis let $I$ be an interval which is either closed or open, and if open need not be bounded. Equation (1) will be called *disconjugate* on $I$ if and only if each solution of (1) has not more than one zero on $I$.

In a recent paper [1] Wintner gave sufficient conditions for the equation (1) to be disconjugate by using the following criterion: Assuming that $p(x)$ and $f(x)$ are continuous functions, (1) is disconjugate on $I$ if and only if there exists on $I$ some function $m = m(x)$ possessing a continuous first derivative which satisfies the inequality

$$m'(x) + p^{-1}(x)m^2(x) \leq -f(x) \tag{2}$$

at every point of $I$. In this paper we shall first give a generalization of this criterion under the lighter conditions which we impose upon the functions $p(x)$ and $f(x)$. With this new criterion we shall generalize a comparison theorem of Hille [2] and extend his criterion. In section 5 we shall discuss the solutions of a self-adjoint differential equation of the third order.

2. **A fundamental criterion.** We shall first prove the following criterion.

Theorem 1. *The equation* (1) *is disconjugate on $I$ if and only if there exists some function $m(x)$ which is absolutely continuous on every closed interval contained in $I$ and satisfies the inequality* (2) *for almost every point of $I$.*

*Proof.* Suppose that the equation (1) is disconjugate on $I$. Then each solution of (1) has not more than one zero on $I$. Using Sturm's separation theorem, it is easy to show that there exists a solution $y_1(x)$ of (1) which does not vanish on $I$. Since $p(x)y_1'(x)$ is equal almost everywhere on $I$ to a function, say $y_2(x)$,

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