UNIQUENESS FOR p-REGULAR MAPPING

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1. **Principal result.** Let a be a positive number. Let Γ denote the circle of radius a about the origin as center in the (x,y)-plane. Let S be the interior of Γ. Let $p(x,y)$ satisfy the following conditions on S and $\bar{S} = S + \Gamma$.

$$p \in C^0(\bar{S}); \qquad p \text{ is real and } > 0 \text{ on } \bar{S}. \tag{1.1}$$

$$p \in C'(S); \qquad p_x,\ p_y \text{ are bounded on } S. \tag{1.2}$$

If $F(z) = u(x,y) + iv(x,y)$, where $z = x + iy$ and u and v are real, is of class C' on S, and if on S,

$$pu_x = v_y, \qquad pu_y = -v_x,$$

then F is p-regular on S and its p-derivative is

$$F'(z) = p^{\frac{1}{2}}u_x + iv_x p^{-\frac{1}{2}}.$$

In [3] the authors proved the following theorem.

Theorem 1. *Let S' be a finite domain whose boundary Γ' is a simple closed rectifiable curve. Let $z^{(1)}$, $z^{(2)}$, $z^{(3)}$ be distinct points on Γ, and let $Z^{(1)}$, $Z^{(2)}$, $Z^{(3)}$ be distinct points on Γ' in the same order on Γ' as the points $z^{(1)}$, $z^{(2)}$, $z^{(3)}$ on Γ. Then there exists a function $F(z)$, continuous on $\bar{S}$, p-regular and with non-vanishing p-derivative on S, such that the transformation $Z = F(z)$ maps $\bar{S}$ onto $S' + \Gamma'$ in a* 1:1 *manner with S, Γ, $z^{(k)}$ corresponding to S', Γ', $Z^{(k)}$.*

The object in the present paper is to complete Theorem 1 by showing that the mapping function obtained in Theorem 1 is unique. We obtain the following somewhat more general result.

Theorem 2. *Let $F_1(z)$, $F_2(z)$ satisfy the following conditions.*

$$F_1,\ F_2 \in C^0(\bar{S}); \qquad F_1,\ F_2 \text{ are } p\text{-regular on } S. \tag{1.3}$$

(1.4) *The transformations $Z = F_1(z)$, $Z = F_2(z)$ map $\bar{S}$ onto the same set, the mapping in each case being* 1:1.

Then $F_1 \equiv F_2$ on $\bar{S}$ if any one of the following four conditions holds.

(1.5) $F_1 = F_2$ *at three (or more) distinct points of* Γ.

(1.6) $F_1 = F_2$ *at one point of* Γ *and at one point of* S.

(1.7) $F_1 = F_2$ *at two distinct points of* S.

(1.8) $F_1 = F_2$ *and* $F_1' = F_2'$ *at one point of* S.

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