CREMONA TRANSFORMATIONS ASSOCIATED WITH THE CHORDS OF A TWISTED CUBIC

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1. **Introduction.** A Cremona space transformation is a one–one relation between generic points P and P' of two spaces S and S', respectively [1; 155]. Such transformations are birational. The spaces S and S' are supposed to exist independently unless the contrary is specified. However, S and S' may be superposed. This gives rise to additional properties of the transformation which are of interest, for example, invariant and involutory elements, self-corresponding elements; in particular, it brings into being the associated complex of lines joining homologous points. Under certain circumstances the complex may reduce to a congruence. In the case of involutorial transformations it is known [1; 181] that if the complex reduces to a congruence this congruence is of the first degree and consists of either:

(i) the lines through a point,

(ii) the lines meeting a line l and a curve of degree m which meets l in $m - 1$ points, or

(iii) the chords of a cubic curve.

Involutions having associated congruences of types (i) and (ii) have been discussed by the author in two previous papers [2], [3]. The present paper is concerned with Cremona transformations, both involutorial and non-involutorial, having associated congruences which are the chords of a twisted cubic. As in the two papers just mentioned the discussion is almost entirely analytic.

2. **Definition of the involution.** Consider a twisted cubic r and a pencil of surfaces

$$| F_{2n+2} | : r^n g_{n^2+8n+4} \ ,$$

of order $2n + 2$, in which the cubic r is contained n times. Through a generic point $P(y)$ there passes a single F of $| F |$, and also through P there is a unique line t belonging to the congruence of chords of r. The line t meets F a second time in a point $Q(x)$, the image of $P(y)$ under the transformation so defined. The residual base curve of $| F |$ has been denoted by g, is of order $n^2 + 8n + 4$, and is considered to be non-composite. It will be shown that r and g are fundamental curves of the involution which is of order $4n + 9$.

3. **Equations of the involution.** Let us take the equation of the twisted cubic r as

$$(1) \qquad x_1 : x_2 : x_3 : x_4 = h^3 : h^2 : h : 1$$

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