THE PLEMELJ THEORY FOR THE CLASS Λ^* OF FUNCTIONS

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1. **Introduction.** Let $f(z)$ be a complex-valued function defined and summable on the rectifiable Jordan curve Γ in the plane of the complex variable z. Define

$$\alpha_1(z) = \frac{1}{2\pi i}\int_\Gamma d\zeta \frac{f(\zeta)}{\zeta - z} \qquad (z \text{ interior to } \Gamma),$$

$$\alpha_2(z) = -\frac{1}{2\pi i}\int_\Gamma d\zeta \frac{f(\zeta)}{\zeta - z} \qquad (z \text{ exterior to } \Gamma),$$

where the direction of integration is to be taken in the counter-clockwise sense. Then $\alpha_1(z)$ and $\alpha_2(z)$ are analytic functions interior and exterior, respectively, to Γ.

Suppose further that arc and chord of Γ are infinitesimals of the same order, that is, for z_1 and z_2 on Γ, there exists a constant A (>1) such that

$$s(z_1, z_2) = \int_{\Gamma(z_1, z_2)} |d\zeta| \leq A\,|z_1 - z_2|.$$

Here $\Gamma(z_1, z_2)$ denotes the shorter arc of Γ which connects z_1 and z_2, or either arc if the two arcs have equal length. We shall continue this notation without further remark. Moreover, if z_0 is on Γ, then $\Gamma(|\zeta - z_0| > \epsilon)$ will denote that portion of Γ exterior to the circular neighborhood of radius ϵ about z_0.

Plemelj [2], Privaloff [3], [4], and Davydov [1] have studied the behavior of $\alpha_1(z)$ and $\alpha_2(z)$ as z approaches Γ, and they have obtained the following results. On curves Γ with the above properties, let $f(z)$ satisfy a Lipschitz condition of order α, $0 < \alpha < 1$. That is, if z_1 and z_2 are arbitrary points on Γ,

$$|f(z_1) - f(z_2)| \leq K\,|z_1 - z_2|^\alpha,$$

where K is a constant independent of z_1 and z_2. If s denotes arc length from a fixed reference point of Γ to a point z, define $F(s) \equiv f(z)$. Since arcs and chords are infinitesimals of the same order, the Lipschitz condition for $f(z)$ is equivalent to the Lipschitz condition

$$|F(s_1) - F(s_2)| \leq K_1\,|s_1 - s_2|^\alpha,$$

with K_1 a suitably chosen constant. Then it is proved that the functions $\alpha_1(z)$, $\alpha_2(z)$ approach uniformly certain boundary functions $f_1(z)$, $f_2(z)$ as z approaches Γ. The functions $f_1(z)$ and $f_2(z)$, defined on Γ, satisfy a Lipschitz

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