MONOTONE INTERIOR DIMENSION-RAISING MAPPINGS

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The purpose of this paper is to demonstrate the existence of monotone interior dimension-raising mappings, and in particular, the existence of a compact one-dimensional continuum M in Euclidean 3-space and a monotone interior mapping of M onto the Hilbert cube H, that is, the topological product of intervals $T_1, T_2, T_3, \cdots$ with the sum of the squares of the lengths of the T_i equal to $k < \infty$. Bing [1] has shown the existence of two-(and higher)-dimensional hereditarily indecomposable continua. Bing's result, together with results of Kelley [2], shows in a very different fashion the existence of monotone interior dimension-raising mappings (of indecomposable continua). Kolmogoroff [3] showed the existence of interior (but not monotone) dimension-raising mappings.

In this paper, the continuum M will be the common part of a sequence $F_1^*, F_2^*, F_3^*, \cdots$, where for each i, F_i is a finite collection of continua and is a refinement of F_{i-1} $(i > 1)$. The mapping will be shown to exist by showing the existence of a continuous collection F of mutually exclusive compact continua filling up M such that F with respect to its elements as points is homeomorphic to H. Each element of F is the common part of a sequence of elements $f_1, f_2, f_3, \cdots$ with f_i containing f_{i+1} and f_i in F_i. The sequence $F_1, F_2, F_3, \cdots$ is to be constructed so that it is similar to a sequence $G_1, G_2, G_3, \cdots$ of collections of compact continua with, for each i, G_i covering H, G_{i+1} a refinement of G_i, and every element of G_i of diameter less than $1/i$.

We give several definitions.

A *refinement* of a collection K is a collection K' such that every element of K' is a subset of an element of K.

A *simple chain* in this paper is a finite collection of 3-cells with a possible ordering of the elements $c_1, c_2, \cdots, c_n$ such that $c_i \cdot c_j$ exists if and only if $| i - j | \leq 1$ and $c_i \cdot c_j$ is a 2-cell if $| i - j | = 1$. Each of the elements of the collection is called a *link* of the chain. We say that a chain contains a point set if the sum of the links of the chain contains the point set.

Two chains are said to be *mutually exclusive* if no link of either intersects a link of the other.

A *connecting link* of two mutually exclusive simple chains e and f is a 3-cell b intersecting exactly one link b_e of e and one link b_f of f such that the diameter of $b + b_e$ is less than the diameter of some link of e and the diameter of $b + b_f$ is less than the diameter of some link of f. It will be understood that the intersection of a connecting link with a link of a chain is a 2-cell if it exists.

In what follows it is possible (but not necessary for the purposes of this paper) to require that the connecting links and the links of all simple chains used here-

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