A PRIMITIVE DISPERSION SET OF THE PLANE

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Wilder in [2; 381] makes the following definitions.

If M is a connected set and D is a subset of M such that $M - D$ is totally disconnected, then D may be called a dispersion set of M. If no proper subset of D is a dispersion set of M, then let us call D a primitive dispersion set of M.

Wilder in [2; 381] also raises the question of the existence of a primitive dispersion set of the plane, E^2. The purpose of this paper is to show the existence of such a set.

Speaking roughly the construction is made as follows. Consider the point set pictured in Figure 1. In each of the A's consider a collection of arcs like those indicated in Figure 2 and let H denote the set of points belonging to arcs of these collections. Similarly, in each B construct arcs as pictured in Figure 3 and let K be the sum of these arcs plus all the boundary points of the B's except the right half of the upper edge of each B and the right end. Then let M be the interior of a simple closed curve and let a be an arc of it. Construct a narrow canal in $M + a$ starting at a and winding toward the rest of the boundary as indicated in Figure 4 and in such a way that a countable set F of mutually exclusive arcs will divide M minus the canal into small domains. Map Figure 1 onto the canal, as indicated in Figure 5, in such a way that the image of A_0 is at the beginning of the canal and the images of H and K separate the canal into small pieces. Call D_1 the sum of the arcs of F and the images of H and K. Each component of $M + a - D_1$ is the interior of a simple closed curve and an arc of its boundary. A construction like that of Figure 4 and a mapping as indicated in Figure 5 can be set up in each of these components and we will call D_2 the sum of these F's plus the images of H and K. By induction then, a

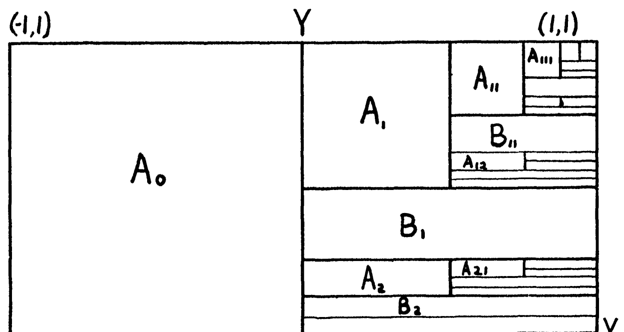

Figure 1

Received January 30, 1952.