A SURVEY OF THE DEVELOPMENT OF THE THEORY OF SCHLICHT FUNCTIONS

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1. Introduction. The study of schlicht functions is one of the most fascinating of all branches of mathematical analysis. From the point of view of conformal mapping the class of analytic schlicht functions is the simplest and most important class of analytic functions. The numerous papers on the subject which appear in the various mathematical journals are evidence of the keen and constant interest shown by researchers in the field. The discovery and application of simple and yet ingenious devices has been a source of abundant results in the theory of schlicht functions. Of course, the introduction of relatively complex variational methods in recent years has been an important step toward the development of a unifying theory. The serious study of schlicht functions began in 1907, so that it is a new field full of opportunity for research. The beginner in this field quickly discovers a great source of inspiration in the contributions of such mathematicians as Koebe, Bieberbach, Gronwall, Golusin, Rogosinski, Robinson, Littlewood, Löwner, Schiffer, Schaeffer, Spencer, and many others. Until 1947 the only historical article on the theory of schlicht functions was a survey by G. M. Golusin, written in Russian. This was translated into English in 1947 by T. C. Doyle, A. C. Schaeffer, and D. C. Spencer. This article can be obtained from the Office of Naval Research, Navy Department, Washington, For a more up-to-date account of the development of the subject the D.C. reader is referred to the excellent book "Coefficient Regions for Schlicht Functions" by A. C. Schaeffer and D. C. Spencer, volume 35 of the Colloquium Publications of the American Mathematical Society, published in 1950.

The purpose of the present article is to present a compact and simple account of the more elementary phases of the subject, avoiding the complicated calculations which fill the pages of the two historical references given above. Necessarily, such an account tends to over-simplify the subject. On the other hand, in the absence of long and tedious calculations, the various results can more easily be recognized and the general trend in the rapid growth of the theory can be more easily observed. To maintain the discussion on an elementary level the construction of variational methods will be avoided and only certain significant The very nature of these results results of these methods will be mentioned. will convince the reader of the power of variational methods in contrast to the type of results obtainable by classical means. The reader will be introduced to a large number of unsolved problems in the theory of schlicht functions. He will also find of great value the bibliography given at the end of this article. In particular, references [10], [47], and [32] are especially valuable from a survey point of view and were heavily relied upon in the writing of this article.

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