CONTINUA AND THEIR COMPLEMENTARY DOMAINS IN THE PLANE. II.

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Introduction. The following theorem is a special case of Theorem 10 of [1].

Theorem A. *If M is a continuum in the plane such that for each three domains R_1, R_2, and R_3 intersecting M there exist three complementary domains of M each intersecting each of the domains R_1, R_2, and R_3, then either M is indecomposable or there is only one pair of indecomposable proper subcontinua of M whose sum is M.*

Also, in [1] the author stated the following corollary to the above theorem. *If M is a continuum in the plane and there is a sequence of distinct complementary domains of M converging to M, then either M is indecomposable or there is only one pair of indecomposable proper subcontinua of M whose sum is M.*

Kuratowski had previously proved the following theorem [3; Theorem 3]. *If the plane continuum M is the boundary of each of three of its complementary domains, then either M is indecomposable or it is the sum of two indecomposable continua.*

In this paper, it is shown that in order that a compact continuum in the plane satisfy the hypothesis of Theorem A, it is necessary and sufficient that either the hypothesis of Kuratowski's theorem be satisfied or the hypothesis of the above corollary be satisfied. (See Theorem 3.) Theorems 4, 5, 6, and 7 concern a continuum M in the plane which possesses the property that for each two domains R_1 and R_2 intersecting M there exist three complementary domains of M each intersecting each of the domains R_1 and R_2.

If P is a point of a continuum M, the set of all points X such that there is a proper subcontinuum of M containing $P + X$ is called a *composant* of M. Janiszewski and Kuratowski have shown [2; 219] that every indecomposable continuum contains uncountably many mutually exclusive composants. Swingle [5] has given the following two definitions: A continuum M is said to be the *finished sum* of a collection G of subcontinua of M if $G^* = M$ and no continuum of G is a subset of the sum of the others. (G^* denotes the sum of the continua of G.) If n is a positive integer and M is a continuum which is the finished sum of n continua but is not the finished sum of $n + 1$ continua, then M is said to be *indecomposable under index n.* In the same paper, Swingle also gave a definition of a *continuum convergible under index n* and obtained the following results. *If $n > 1$ and the continuum M is convergible under index n, then* (1) *there exist n*

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