REPRESENTATION OF COMPLETE SYSTEMS OF FUNCTIONS

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Introduction. It has been shown by Gelfand [3], [4] that a Banach algebra over the reals in which every element x satisfies (i) $e + x^2$ always has an inverse, where e is the unit element, and (ii) $|| x^2 || = || x ||^2$, is isomorphic and isometric to the ring of continuous functions on a suitable compact Hausdorff space. Since he was concerned with an abstract Banach algebra, his representation for this space is necessarily quite general; indeed, it is in terms of a space of maximal ideals of the Banach algebra. One would expect, then, that for a particular Banach algebra a more direct characterization could be obtained. It is the purpose of this paper to find such a representation when the Banach algebra in question is a complete system of bounded real-valued functions on a topological space S. The representation, in terms of lattice, instead of ring, operations, gives the space as a Wallman [11] space associated with the distributive lattice of spectral sets of the system. In this connection, it should be noted that Representation Theorem II generalizes a result of Higman [6] concerning the continuous functions on a normal space (Corollary 1.4.1). The representation theorems are the substance of Part I. In Part II, these results are applied to give a representation for the Baire functions of class α on S.

Notation. Throughout this paper the following notation will be consistently used. S denotes the basic topological space, and s a collection of bounded real-valued functions on S; x, y are points of S, while f, g, h are members of s. $C(S)$ denotes the collection of all bounded real-valued continuous functions on S, while $K(S)$ denotes the collection of all bounded real-valued functions on S. Subsets of S are denoted by capital Latin letters A, B, C, P, Q, collections of subsets of S by capital Greek letters, while if such a collection is a distributive lattice, the Wallman space associated with it is denoted by the corresponding capital German letter. The small German letters $\mathfrak{m}$, $\mathfrak{n}$, $\mathfrak{p}$, $\mathfrak{q}$ are minimal (dual) ideals of a lattice, or points of a Wallman space. Finally, to avoid confusion, lattice operations are indicated by the pointed symbols $\wedge$, $\vee$, $\leq$; while set operations are indicated by the corresponding rounded symbols $\cap$, $\cup$, $\subseteq$.

Part I. The Representation Theorems

1.1 **Definitions and preliminary results.** Following Hausdorff [5], a complete system of functions is defined in the following way.

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