REMARKS ON THE CLAYTOR IMBEDDING THEOREM

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It was shown by Kuratowski [4] that a Peano space (that is, a compact, metric, locally connected continuum) is imbeddable in a 2-sphere if it contains only a finite number of simple closed curves and contains neither of the two "primitive skew graphs" defined as follows: (I) P_1 consists of the points p_1, $p_2, \cdots, p_5$ and the arcs $(p_i p_j)$, where $1 \leq i < j \leq 5$, and where different arcs $(p_i p_j)$ intersect only at their end-points. (II) P_2 consists of the points p_1, p_2, p_3 and q_1, q_2, q_3 and the arcs $(p_i q_j)$, where different arcs $(p_i q_j)$ intersect only at their end-points. It was shown by Claytor [2] that if S is a Peano space which has no cut-point and which contains no set homeomorphic to P_1 or P_2, then S is imbeddable in a 2-sphere. The object of the present note is to give a proof of Claytor's theorem, based on a brick partitioning theorem due to R. H. Bing.

A *grille decomposition* of a Peano space is a finite collection G of mutually exclusive connected, uniformly locally connected open sets, such that $\overline{G^*}$ is S. (G^* denotes the sum of the elements of G.) The *mesh* of G is the maximum of the diameters of its elements. A complete sequence of grille decompositions of S is a sequence of $G_1, G_2, \cdots$ of grille decompositions of S, such that for each i, G_{i+1} is a refinement of G_i, and such that for any positive number ϵ, only a finite number of the G_i's have mesh greater than ϵ. Given a grille decomposition G of S, the *1-nerve of* G is an abstract graph whose vertices correspond to the elements of G, such that two vertices are joined by a (unique) edge if and only if the corresponding elements of G have a boundary point in common. If G is a grille decomposition of S, such that for each g, g' of G, the interior of $\overline{g} \cup \overline{g}'$ is uniformly locally connected, then we say that G is a brick partitioning of S [1]. It has been shown by Bing [1; Theorem 8] that every Peano space has a complete sequence of brick partitionings.

Let S be a space satisfying the hypothesis of Claytor's imbedding theorem, and let $G_1, G_2, \cdots$ be a complete sequence of brick partitionings of S. Let H_1 be the 1-nerve of G_1.

Lemma. *H_1 is imbeddable in a 2-sphere.*

Proof. For each element g_i of G_i, let p_i be a point of g_i; and for each g_i, g_j for which $\overline{g_i} \cap \overline{g_j} \neq 0$, let p_{ij} be a point of $\overline{g_i} \cap \overline{g_j}$ which does not belong to the closure of $S - (g_i \cup g_j)$. Since each g_i is locally connected at each point of its boundary, each set $g_i \cup p_{ij}$ is locally connected, whence it follows from a theorem of Moore [7; 86] that there is an arc $p_i p_{ij}$ which lies, except for p_{ij}, wholly in g_i. (While $g_i \cup p_{ij}$ is neither compact not locally compact, it satisfies

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