THE MAP EXCISION THEOREM

By A. D. Wallace

In this note we prove the following:

Map excision theorem. *Let X and Y be fully normal and A and B closed. Let*

$$f : (X, A) \to (Y, B)$$

be a closed map such that f takes $X - A$ topologically onto $Y - B$. Then

$$f^* : H^p(Y, B) \approx H^p(X, A).$$

Here H^p denotes the Alexander-Kolmogoroff cohomology group (Spanier [5]). The precise parentage of this proposition is not known to me. It was formulated less completely by H. Cartan [1] for what amounts to the Čech theory.

I am indebted to N. E. Steenrod for a proof in case X and Y are compact Hausdorff. It then follows from the continuity theorem (Spanier [5]). A different version (X, Y locally compact cocycles with compact supports) is in Leray [4]. See also the forthcoming book of Eilenberg and Steenrod [2]. The particular interest that our version has is this: It is valid in any metric space and in any σ-compact, locally compact Hausdorff space *without using compact cocycles.* Its validity in closed subsets of linear locally convex spaces is conjectural.

We shall need the extension and reduction theorems [6]. The proofs given here are essentially those of [6]. We note that these theorems follows from the continuity theorem if X is compact Hausdorff, but we do not need this.

Extension theorem. *Let X be fully normal and X_0 , A closed. If $e \in H^p(A, A \cap X_0)$ then, for some open set N containing A, e can be extended to the group $H^p(\bar{N}, \bar{N} \cap X_0)$.*

Reduction theorem. *Let X be fully normal and X_0 and A closed. If $e \in H^p(X, X_0)$ and e is taken by the natural homomorphism into the zero of $H^p(A, A \cap X_0)$ then, for some open set N containing A, e is taken by the natural homomorphism into the zero of $H^p(\bar{N}, \bar{N} \cap X_0)$.*

If $\mathfrak{U}$ is an open cover of X and $U_0 \in \mathfrak{U}$ let

$$U_0^* = \cup\{U \mid U \in \mathfrak{U} \quad \text{and} \quad U \cap U_0 \neq \square\}.$$

Define

$$\mathfrak{U}^* = \{U^* \mid U \in \mathfrak{U}\}.$$

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